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Lepton-number violation and right-handed neutrinos in Higgs-less effective theories

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Following previous work, we identify a symmetry $S_{\rm nat}$ that generalizes the concept of custodial symmetry, keeping under control deviations from the Standard Model (SM). To realize $S_{\rm nat}$ linearly, the space of gauge fields has to be extended. Covariant constraints formulated in terms of spurions reduce $S_{\rm nat}$ back to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. This allows for a covariant introduction of explicit $S_{\rm nat}$-breaking parameters. We assume that $S_{\rm nat}$ is at play in a theory of electroweak symmetry-breaking without a light Higgs particle. We describe some consequences of this assumption, using a non-decoupling effective theory in which the loop expansion procedure is based on both momentum and spurion power counting, as in Chiral Perturbation Theory. A hierarchy of lepton-number violating effects follows. Leading corrections to the SM are non-oblique. The effective theory includes stable light right-handed neutrinos, with an unbroken $\mathbb{Z}_2$ symmetry forbidding neutrino Dirac masses. ν_R contribution to dark matter places bounds on their masses.

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I. INTRODUCTION

In this paper, we detail a systematic low-energy expansion procedure applied to scenarios of electroweak symmetry-breaking (EWSB) without a Higgs particle, and without any other so far undiscovered particle below the TeV scale (except maybe right-handed neutrinos). The aim is to construct an effective theory in which the smallness of deviations from the Standard Model (SM) would be controlled by a symmetry, i.e. *technically natural* [1]. In its minimal version, the theory only contains $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Yang-Mills fields and chiral fermions coupled to three Goldstone bosons (GBs) which disappear from the spectrum, resulting in three of the vector fields acquiring a mass. All these degrees of freedom are light compared to the scale $\Lambda_{\rm w}$

$$\Lambda_{\rm w} \simeq 4\pi v \simeq 3\,\mathrm{TeV}. \tag{1.1}$$

Under these circumstances, the theory is not renormalizable in the usual sense, i.e. in powers of coupling constants. Instead, it has to be defined, renormalized (and unitarized) in powers of external momenta, generalizing the example and techniques of Chiral Perturbation Theory (χPT) [2–4]. This presumes a self-consistent infrared power counting which would make it possible to order operators in the lagrangian $\mathcal{L}_{\rm eff}$ as well as loops according to their importance in the low-energy limit. It is then understood that each operator allowed by the symmetries has to be included into $\mathcal{L}_{\rm eff}$ at the order corresponding to its infrared dimension. The relevant infrared power-counting for operators and Feynman diagrams is reviewed in Sections III A, III B and in Appendix A.

The essential new ingredient concerns the symmetry $S_{\rm nat}$ of the lagrangian underlying the low-energy effective theory (LEET). We require $S_{\rm nat}$ to be sufficiently large as to force the leading $\mathcal{O}\left(p^2\right)$ order of the LEET to coincide with the tree-level Higgs-less vertices of the SM in the limit of vanishing fermion masses. It is more easy to motivate this requirement than to find its appropriate mathematical implementation:

i) First, a LEET based on $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as the maximal symmetry group —the case considered in the past [5–13] — does *not* fulfill the above requirement: indeed, there are several non-standard and unobserved $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-invariant vertices that appear at the leading order $\mathcal{O}\left(p^2\right)$ [14]. They represent a priori unsuppressed tree-level contributions to the S [6, 15] and T [16] parameters, non-standard couplings of left-handed fermions to vector bosons [5, 17], or introduce couplings of right-handed fermions to the $W^\pm$. They are discussed in Section III C where it is shown that a LEET based on such a small symmetry group would in addition allow a large Majorana mass for left-handed neutrinos, and large $\mathcal{O}\left(p^2\right)$ lepton-number violating (LNV) vertices in general.

ii) Next, if the absence of non-standard $\mathcal{O}\left(p^2\right)$ vertices is to be explained by a *higher symmetry group* $S_{\rm nat}$

$$S_{\rm nat} \supset S_{\rm red} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{1.2}$$

the action of $S_{\rm nat}/S_{\rm red}$ on the GBs and on the original SM set of gauge fields, must be non-linear. This complicates the task of inferring $S_{\rm nat}$ from the SM lagrangian. The nature of this problem can be illustrated by the following example, given here for a pedagogical purpose: suppose that one deduced from $\pi\pi$ scattering experiments the fol-

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[1] Throughout this paper, we use the words *(technically) natural* and *naturally* as follows: it is technically natural for a parameter to be small if there is a symmetry related to the limit in which it is sent to zero [1].

lowing effective lagrangian [2]

$$\mathcal{L}^{\text{eff}}_{\pi\pi} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{1}{2f^2}\left(1-\frac{\vec{\pi}^2}{f^2}\right)^{-1} \times\left(\vec{\pi}\cdot\partial_\mu\vec{\pi}\right)\left(\vec{\pi}\cdot\partial^\mu\vec{\pi}\right) + \mathcal{O}\left(p^4\right). \quad (1.3)$$

We could then ask why at the leading order $\mathcal{O}\left(p^2\right)$, various terms allowed by the linear isospin symmetry $O\left(3\right)$ are actually absent. The task would then be to rediscover the $O\left(4\right)/O\left(3\right)$ non-linearly realized chiral symmetry. It is known that the problem is greatly simplified by the introduction of an additional field σ such that the action of $O\left(4\right)$ on the enlarged manifold $\left(\sigma,\vec{\pi}\right)$ is linear. The field σ is subject to the $O\left(4\right)$-invariant constraint

$$\sigma^2 + \vec{\pi}^2 = f^2, \quad (1.4)$$

and the lagrangian (1.3) is equivalently rewritten as

$$\mathcal{L}^{\text{eff}}_{\pi\pi} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right) + \mathcal{O}\left(p^4\right). \quad (1.5)$$

This lagrangian does not describe any interaction unless the constraint (1.4) is applied.

iii) We show in Section IV that a similar procedure exists in the case of Higgs-less vertices of the SM, leading to an explicit description of its "hidden symmetry" S_{nat}. This higher symmetry $S_{\text{nat}} \supset S_{\text{red}}$ can be linearized by adding a set of nine auxiliary gauge fields to the original four present in the SM. The additional gauge fields are no more physical than the σ field of the previous example. At the end, they are eliminated by S_{nat}-invariant constraints akin to (1.4). Before these constraints are applied, the lagrangian of the theory at $\mathcal{O}\left(p^2\right)$ consists of two decoupled sectors, as in equation (1.5): a) the symmetry-breaking sector containing three GBs together with six gauge connections of the gauged spontaneously-broken $\text{SU}\left(2\right)_{\Gamma_L}\times\text{SU}\left(2\right)_{\Gamma_R}$ symmetry *and* b) the unbroken $\text{SU}\left(2\right)_{G_L}\times\text{SU}\left(2\right)_{G_R}\times\text{U}\left(1\right)_{B-L}$ gauge theory with the $L\leftrightarrow R$ symmetric coupling of local left and right isospin to chiral fermion doublets [18].

S_{nat} may be defined as the maximal local linear symmetry the theory could have if the symmetry breaking sector and gauge/fermion sector are decoupled. In the present case, one has

$$S_{\text{nat}} = \left[\text{SU}\left(2\right)\times\text{SU}\left(2\right)\right]^2\times\text{U}\left(1\right)_{B-L}, \quad (1.6)$$

and the unconstrained theory contains thirteen gauge fields: four are physical, whereas the nine remaining "σ-type fields" extend the custodial symmetry, originally related to the right-isospin group [16, 19]. The S_{nat}-invariant constraints eliminate the nine redundant "σ-type fields", reduce the *linear symmetry* S_{nat} to its electroweak subgroup S_{red} and induce couplings between the symmetry-breaking and gauge/fermion sectors. At this stage, $W^\pm$ and Z^0 become massive, whereas all fermions remain massless. In this way one recovers all Higgs-less vertices of the SM. The symmetry $S_{\text{nat}}/S_{\text{red}}$ is realized non-linearly and hidden similarly to the $O(4)/O(3)$ symmetry of the non-linear σ-model hidden in the pion lagrangian (1.3). The main effect of S_{nat} is the elimination of all non-standard $\mathcal{O}\left(p^2\right)$ vertices.

iv) The analogy with the non-linear σ-model is however not complete, and this fact makes the problem even more interesting. Whereas the constraint (1.4) concerns spin-0 fields, the constraints in the electroweak case operate with gauge fields: a gauge configuration $G_\mu \in s_{\text{nat}}$ [2] satisfies the constraint if there exists a gauge transformation Ω such that the image H_μ of G_μ by Ω belongs to the sub-algebra s_{red}

$$G_\mu \overset{\Omega}{\longmapsto} H_\mu \in s_{\text{red}} \subset s_{\text{nat}}. \quad (1.7)$$

Covariance of (1.7) implies that Ω itself should transform in a definite (non-linear) way under S_{nat}. The analysis of this constraint parallels that of the spontaneous symmetry breaking $S_{\text{nat}} \longrightarrow S_{\text{red}}$, and of the corresponding Goldstone theorem. There exist $n = \dim s_{\text{nat}} - \dim s_{\text{red}}$ scalar objects —referred to as *spurions* — that live in the coset space $S_{\text{nat}}/S_{\text{red}}$ and transform under S_{nat} as GBs would do. However, we shall see that the constraint (1.7) implies that spurions have vanishing covariant derivatives and, consequently, in contrast to GBs, the spurions do not propagate and do not generate mass terms for vector fields either. There exists a "standard gauge" in which spurions reduce to a set of constants. In the actual case of the group S_{nat} (1.6), the nine spurions reduce to three constants, denoted ξ, η and ζ. This reflects the structure of the coset space which, in this case, is a product of three SU (2) groups.

Spurions allow to keep track of the original symmetry S_{nat}, even if the latter is explicitly broken. In particular, ξ, η and ζ can be considered as small expansion parameters describing perturbatively the explicit breaking of S_{nat}. From this point of view, spurions play a role similar to quark masses in χPT: they allow for a classification of explicit symmetry-breaking operators under the assumption that such effects are small. As in the case of quark masses, the smallness of ξ, η and ζ is protected by the symmetry. The complete LEET invariant under S_{nat} should be defined as a double expansion: in powers of momenta and in powers of spurions. The LEET at leading order coincides with the Higgs-less vertices of the SM, used at tree-level. The non-standard $\mathcal{O}\left(p^2\right)$ vertices mentioned above now reappear as S_{nat}-invariant operators explicitly containing spurions, i.e. suppressed by powers of the parameters ξ, η and ζ.

Such spurions have been introduced in [14, 20], where their existence and properties have been postulated and justified *ad hoc*. In the present work, a deeper group-theoretical insight into the origin and role of spurions is

[2] Small letters denote the algebra of a group.

established and presented in a self-contained way in Section IV. Spurions thus appear as unavoidable elements of a LEET in which the dominance of SM vertices is a consequence of a higher symmetry $S_{\rm nat}$ [3].

In Sections II, III and IV, the origin of spurions and of the covariant reduction of the symmetry $S_{\rm nat}$ to $S_{\rm red} = {\rm SU}(2)_L \times {\rm U}(1)_Y$ is discussed in details. The emphasis is put from the beginning on the special status of the lepton-number violating (LNV) sector in a non-decoupling LEET. The following Sections V, VI and VII are devoted to observable consequences of the spurion formalism, starting with the Dirac masses of charged fermions in Section V A. Special attention is paid to the spurion effects which —by power-counting arguments— are expected to contribute before loops. These are genuine effects beyond the SM. Section VI contains a complete list of such next-to-leading (NLO) effects in the lepton-number conserving sector. These effects merely involve universal non-standard couplings of fermions to massive vector bosons. They are suppressed by the spurions ξ and η. The oblique corrections only appear at the NNLO, together with loops, and consequently, they are even more suppressed. The phenomenological analysis of the NLO is underway [22].

In Section IV it is shown that the existence of $\Delta L = 2$ LNV vertices is a necessary consequence of the reduction $S_{\rm nat} \to S_{\rm red}$: one of the resulting spurions carries two units of the $B-L$ charge. All LNV effects, in particular the Majorana masses of both left-handed and right-handed neutrinos, are suppressed by the corresponding spurion strength ζ^2. Since the symmetry group $S_{\rm nat}$ includes the right isospin group as the origin of the custodial symmetry, the LEET necessarily contains three species of *light* right-handed neutrinos. The usual see-saw mechanism [23–26] is not efficient. Instead, we assume in Section V that a non-anomalous $\mathbb{Z}_2$ subgroup of the flavor symmetry, which we call ν_R sign-flip symmetry, remains unbroken; it forbids neutrino Dirac masses. In this manner, the right-handed neutrinos decouple from the other fermions. There are three different possibilities for the introduction of the $(B-L)$-breaking parameter ζ^2, resulting in different estimates for the ratio of right- to left-handed neutrinos masses. Two of them seem to be allowed by cosmological observations constraining the contribution of light and stable sterile right-handed neutrinos to dark matter (DM), as is discussed in Section VIII. Section VII focuses on the relative importance of indirect and direct LNV contributions to the process $W^-W^- \longrightarrow e^-e^-$, a building block for neutrino-less double beta decay ($0\nu2\beta$) and other $\Delta L = 2$ processes. We give our conclusions in Section IX.

[3] In models based on reduction of extra-dimensional theories (see for the Higgs-less case [21] and references therein), the suppression of certain vertices need not result from extra symmetries, but rather from the locality along the extra dimension. This suppression itself can be improved by considering a curved extra-dimension.

II. LNV IN THEORIES WITH ELEMENTARY SCALARS

Before we turn to LNV in Higgs-less effective theories, we recall the fate of this accidental symmetry in the SM.

A. Right isospin and corresponding notations

The complex Higgs doublet of the SM transforms under weak ${\rm SU}(2)_L$ gauge transformations G and the ${\rm U}(1)_Y$ gauge function α_Y as

$$\varphi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix} \longmapsto G e^{-{\rm i}\frac{\alpha_Y}{2}} \varphi. \tag{2.1}$$

For further convenience, we display the custodial symmetry [19], using a two-by-two matrix Φ [27]

$$\begin{aligned} \Phi &\equiv (\varphi_c, \varphi) \\ &= \begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix} \longmapsto G\Phi e^{{\rm i}\frac{\tau^3}{2}\alpha_Y}, \end{aligned} \tag{2.2}$$

where the conjugate $\varphi_c \equiv {\rm i}\tau^2\varphi^* \longmapsto G e^{{\rm i}\frac{\alpha_Y}{2}}\varphi_c$ of the Higgs doublet φ has been introduced. The most general renormalizable lagrangian involving Φ and invariant under ${\rm SU}(2)_L \times {\rm U}(1)_Y$ is

$$\mathcal{L}_{\rm Higgs} = \frac{1}{2}\left\langle D_\mu\Phi^\dagger D^\mu\Phi\right\rangle - \frac{\lambda}{4}\left(\left\langle\Phi^\dagger\Phi\right\rangle - v^2\right)^2, \tag{2.3}$$

where $\langle A\rangle \equiv {\rm Tr}\, A$ and $D_\mu\Phi = \partial_\mu\Phi - {\rm i}gG_\mu\Phi + {\rm i}g'\Phi b_\mu\frac{\tau^3}{2}$.

In the limit $g' = 0$, the lagrangian $\mathcal{L}_{\rm Higgs}$ (2.3) is invariant under global ${\rm SU}(2)_R$ transformations acting from the right on Φ [4]. We stress that, when $g' \neq 0$, only the third component of the ${\rm SU}(2)_R$ group is gauged. This ${\rm SU}(2)_R$ is the group of right isospin transformations, as can be seen when writing the transformations of the fermions (depending on their baryon and lepton numbers B and L respectively)

$$\chi_L = \frac{1-\gamma_5}{2}\chi \longmapsto G e^{-{\rm i}\frac{B-L}{2}\alpha_Y}\chi_L, \tag{2.4}$$

$$\chi_R = \frac{1+\gamma_5}{2}\chi \longmapsto e^{-{\rm i}\frac{\tau^3}{2}\alpha_Y} e^{-{\rm i}\frac{B-L}{2}\alpha_Y}\chi_R, \tag{2.5}$$

where χ consists of two Dirac spinors arranged in a column, and will be denoted by q in the case of quarks ($B-L = 1/3$) and ℓ for the case of leptons ($B-L = -1$).

[4] As pointed out in the introduction, the $S_{\rm nat}$ symmetry can be viewed as defining this custodial symmetry (and its extension to the left-handed non-abelian sector) without requiring the gauge couplings to be zero.

Note that this writing does not imply the existence of a right-handed neutrino, since such a field is not charged under the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry. On the other hand, (2.2) shows that the Higgs doublet has a non-zero value for the third component of the right isospin: $T_R^3 = 1/2$, but vanishing $(B-L)$, as should be. In fact, the right isospin group is explicitly broken not only by gauging the hypercharge, but also by the Yukawa terms for the fermions: invariance under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ allows for different masses for the two components of a fermion doublet

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\sum_{i,j=1}^{3}\left(\overline{q_L^i}\Phi\begin{pmatrix} y_{ij}^u & 0 \\ 0 & y_{ij}^d\end{pmatrix} q_R^j\right) \\ &\quad -\sum_{i,j=1}^{3}\left(\overline{\ell_L^i}\Phi\begin{pmatrix} 0 & 0 \\ 0 & y_{ij}^e\end{pmatrix}\ell_R^j\right) + \text{h.c.}\end{aligned} \quad (2.6)$$

The sum running over $i,j = 1,\cdots,3$ in the above equation corresponds to the three generations. Adding the gauge-invariant kinetic terms for the gauge fields and the fermions, one recovers the lagrangian for the SM. As an outcome of the symmetry-breaking mechanism, the generator corresponding to the unbroken $\mathrm{U}(1)_Q$ subgroup can be expressed as [18]

$$Q = T_L^3 + T_R^3 + \frac{B-L}{2}, \quad (2.7)$$

where the third component of the left isospin $\mathrm{SU}(2)_L$ and right isospin $\mathrm{SU}(2)_R$, respectively T_L^3 and T_R^3, appear. This formula also applies to the Higgs doublet, but it does not imply the presence in the theory of both left and right isospin gauge fields.

B. The unique mass-dimension five effective operator

The renormalizable SM lagrangian is defined to consist of all operators of mass-dimension $d_M \leqslant 4$ and invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ that can be built with the fields introduced above. These fields necessarily involve elementary Higgs scalars, and they transform linearly with respect to the symmetry group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Taking the view that the SM is an effective theory, which has to be completed at higher energies, one augments the renormalizable SM lagrangian with effective operators of dimension higher than four, constructed with the same fields, and respecting the same symmetries

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d_M>4}\frac{1}{\Lambda^{d_M-4}}\mathcal{O}_{d_M}. \quad (2.8)$$

The effective operators $\mathcal{O}_{d_M}$ describe the effects of new physics beyond the SM. They are "irrelevant" at low energies: they are suppressed by a dimensionful scale Λ, related to that at which new physics appears. Due to the property of renormalizability, the separation of the lagrangian in two distinct parts —the SM lagrangian on one side, and the effective operators on the other— is preserved in loop calculations. In particular, there is no theoretical inconsistency in assuming that the dimensionful scale Λ is arbitrarily large, in which case the effects of new physics become vanishingly small —hence the qualification "decoupling".

The only mass-dimension five $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-invariant effective operator that can be built with the fields of the SM, and which is therefore suppressed by the smallest power of Λ, is the following lepton number violating operator [28]

$$\mathcal{L}_{\text{Majorana}L} = -\frac{1}{\Lambda}\sum_{i,j=1}^{3} c_{ij}\overline{\ell_L^i}\Phi\tau^+\Phi^\dagger\left(\ell_L^j\right)^c + \text{h.c.} \quad (2.9)$$

We have used the following definition for the conjugate of a doublet of left-handed leptons

$$(\ell_L)^c = \mathrm{i}\tau^2 C\overline{(\ell_L)}^T = \begin{pmatrix} (e_L)^c \\ -(\nu_L)^c\end{pmatrix}, \quad (2.10)$$

where C is the charge conjugation matrix, defined to satisfy $C^{-1}\gamma_\mu C = -\gamma_\mu^T$.

The effective operator (2.9) encodes the low-energy consequences of an unknown mechanism generating masses for the left-handed neutrinos. The description by an effective operator is useful if the scale Λ is large enough (with eigenvalues of order unity for the matrix c_{ij}) compared to the vacuum expectation value v of the Higgs field. One concludes that the left-handed neutrinos have Majorana masses of order v^2/Λ. Provided Λ is large enough, this may well be much smaller than the masses of the charged fermions, which are of order v times a Yukawa coupling [5]. The see-saw mechanism [23–26] would provide one possible dynamical origin for the term (2.9). In this case, Λ is given in terms of the right-handed Majorana masses.

III. AT WHICH CHIRAL ORDER DOES LNV APPEAR IN THE HIGGS-LESS EFFECTIVE THEORY?

In the case of Higgs-less EWSB, we do not have a renormalizable theory to start with, and the framework of decoupling effective theories (2.8) is not suitable. In the alternative framework of *non-decoupling* effective theories, all operators respecting the symmetries must be included in the effective lagrangian, but they are no longer ordered according to their mass-dimension d_M as in (2.8): one

[5] We will not get into the problem of accounting for the six orders of magnitude between the electron and top quark masses, which is part of the flavor problem.

must use instead the infrared (or chiral) dimension $d_{\rm IR}$ reflecting their behavior in the low-energy limit $p \to 0$. The effective lagrangian is expressed as

$$\mathcal{L}_{\rm eff} \;=\; \sum_{d_{\rm IR}\geqslant 2} \mathcal{L}_{d_{\rm IR}}\,, \qquad (3.1)$$

where in the limit of small momenta p, the operators of $\mathcal{L}_{d_{\rm IR}}$ behave as

$$\mathcal{L}_{d_{\rm IR}} \;=\; \mathcal{O}\left(p^{d_{\rm IR}}\right). \qquad (3.2)$$

The loops are renormalized order by order in the low-energy expansion (3.1). We summarize the rules of the *infrared power-counting* underlying the expansion (3.1) and the corresponding order-by-order renormalization. Afterwards, we ask at which place of the expansion (3.1) LNV appears for the first time.

A. Infrared power counting

In the minimal Higgs-less theory, the low-energy description of the symmetry-breaking sector comprises only the three GBs that are eaten to give masses to the gauge bosons $W^{\pm}$ and Z^0. In addition, there are chiral fermions χ_L, χ_R. As in χPT, the three GBs $\vec{\pi}\,(x)$ are collected in a matrix $\Sigma\,(x) \in {\rm SU}\,(2)$, and they transform non-linearly with respect to ${\rm SU}\,(2)_L \times {\rm U}\,(1)_Y$ (compare (2.2))

$$\Sigma\,(x) \;\equiv\; {\rm e}^{\frac{\rm i}{f}\vec{\pi}\,(x)\cdot\vec{\tau}} \;\longmapsto\; G\,(x)\,\Sigma\,(x)\,{\rm e}^{{\rm i}\alpha_Y(x)\frac{\tau^3}{2}}. \qquad (3.3)$$

All particles included in the LEET should have their masses protected by a symmetry; masses then only come in with an explicit power of expansion parameters. A crucial ingredient for the non-decoupling effective theory is therefore technical naturalness [1]: there should be a limit in the parameter space, in which all particles become massless, and the symmetry of $\mathcal{L}_{\rm eff}$ is increased. The LEET involves a systematic expansion around that limit. The original discussion of power counting for the GB sector in χPT can be found in [2–4]. Generalizations to include other degrees of freedom have already been considered in [10, 29] for the case of gauge fields, and in [10, 12] for the case of chiral fermions.

1. *Goldstone bosons*

Due to the non-linear transformation (3.3), the GB fields $\vec{\pi}$ carry no infrared dimension, i.e. $\vec{\pi} = \mathcal{O}\,(1)$. Their physical mass-dimension one is compensated by the dimensionful constant f which represents an intrinsic scale of the theory and is not affected by the low-energy limit: in χPT, it coincides with the pion decay constant $f_\pi \simeq 92.4\,{\rm MeV}$, whereas in the case of EWSB, f replaces the Higgs vacuum expectation value $f \simeq 250\,{\rm GeV}$. Hence, in the low-energy limit

$$\Sigma \;=\; \mathcal{O}\,(1)\,, \qquad (3.4)$$
$$D_\mu\Sigma \;=\; \mathcal{O}\,(p)\,, \qquad (3.5)$$

where D_μ is the covariant derivative. The lowest-order contribution of GBs to $\mathcal{L}_{\rm eff}$ takes the form

$$\mathcal{L}_{\rm GB} \;=\; \frac{f^2}{4}\left\langle D_\mu\Sigma^\dagger D^\mu\Sigma\right\rangle \;=\; \mathcal{O}\left(p^2\right). \qquad (3.6)$$

Estimating loop contributions, one infers that the LEET should be applicable for momenta [30, 31]

$$p \;\ll\; 4\pi f \;\simeq\; 3\,{\rm TeV}\,. \qquad (3.7)$$

2. *Gauge fields*

In order to ensure that the vector bosons W_μ and Z_μ be naturally light, one has to treat them as weakly-coupled gauge fields

$$gG_\mu \;\in\; {\rm su}(2), \quad g'b_\mu \;\in\; {\rm u}\,(1)\,, \qquad (3.8)$$

acquiring their masses via the Higgs mechanism (without a Higgs particle). We stress that this is a necessity once one requires a low-energy power-counting involving these vector fields [10, 32, 33]. At leading order, M_W and M_Z can be directly inferred from (3.6). The consistency of the low-energy expansion requires the squared masses to be counted as $\mathcal{O}\left(p^2\right)$

$$M_W^2 \;=\; \frac{g^2}{4}f^2 \;=\; \mathcal{O}\left(p^2\right), \qquad (3.9)$$
$$M_Z^2 \;=\; \frac{g^2+g'^2}{4}f^2 \;=\; \mathcal{O}\left(p^2\right). \qquad (3.10)$$

Since f^2 is a fixed scale, one must count the gauge couplings as

$$g^2 \;\simeq\; \frac{4M_W^2}{f^2} \;=\; \mathcal{O}\left(p^2\right), \qquad (3.11)$$
$$g'^2 \;\simeq\; \frac{4}{f^2}\left(M_Z^2 - M_W^2\right) \;=\; \mathcal{O}\left(p^2\right). \qquad (3.12)$$

Hence, in the low-energy limit, gauge couplings must be considered as vanishing proportionally to external momenta. Since the covariant derivative $D_\mu\Sigma$ (3.5) involves gauge connections gG_μ and $g'b_\mu$, (3.11-3.12) in turn imply that the infrared dimension of canonically normalized gauge fields G_μ and b_μ vanishes

$$G_\mu \;=\; \mathcal{O}\,(1)\,, \qquad (3.13)$$
$$b_\mu \;=\; \mathcal{O}\,(1)\,. \qquad (3.14)$$

In the field strength $G_{\mu\nu}$, both the derivative and the non-linear terms thus count as $\mathcal{O}\,(p)$

$$G_{\mu\nu} \;=\; \partial_\mu G_\nu - \partial_\nu G_\mu - {\rm i}g\,[G_\mu, G_\nu] \;=\; \mathcal{O}\,(p)\,. \qquad (3.15)$$

Consequently, in the Yang-Mills lagrangian

$$\mathcal{L}_{\rm YM} \;=\; -\frac{1}{2}\left\langle G_{\mu\nu}G^{\mu\nu}\right\rangle \;=\; \mathcal{O}\left(p^2\right), \qquad (3.16)$$

the kinetic term *and* both trilinear and quartic couplings are counted as $\mathcal{O}\left(p^2\right)$. Hence the low-energy expansion preserves gauge invariance order by order, in contrast with the more familiar expansion in powers of coupling constants *only*, used in renormalizable theories. Finally, since $M_W, M_Z = \mathcal{O}(p)$, the massive gauge boson propagators exhibit the homogeneous low-energy behavior $\mathcal{O}\left(p^{-2}\right)$.

3. *Chiral fermions*

As was the case for GBs and gauge fields, the infrared dimension of chiral fermion fields is one unit less than their physical dimension

$$\chi_{R,L} \;=\; \mathcal{O}\left(p^{1/2}\right). \tag{3.17}$$

This corresponds to the expected low-energy behavior of fermion bilinears $\overline{\chi}\Gamma\chi = \mathcal{O}(p)$. In particular, the fermion kinetic term contributes to the lagrangian as

$$\mathcal{L}_f \;=\; \mathrm{i}\overline{\chi}\gamma^\mu D_\mu\chi \;=\; \mathcal{O}\left(p^2\right), \tag{3.18}$$

similarly to GBs (3.6) and to gauge fields (3.16). On the other hand, the fermion mass term is counted as

$$\mathcal{L}_{\rm mass} \;=\; -m\overline{\chi}\chi \;=\; \mathcal{O}(mp). \tag{3.19}$$

Unless the fermion mass m is suppressed *at least* as $\mathcal{O}(p)$, i.e.

$$m \;=\; \mathcal{O}(p^n),\; n \geqslant 1, \tag{3.20}$$

the low-energy behavior $\mathcal{O}\left(p^{-1}\right)$ of the fermion propagator is destroyed. We shall return to this potential problem shortly.

The above discussion can be summarized as follows. A local operator/interaction vertex $\mathcal{O}$ built from GBs Σ, gauge fields G_μ and b_μ, and from fermions $\chi_{L,R}$ carries the infrared dimension

$$d_{\rm IR}[\mathcal{O}] \;=\; n_\partial[\mathcal{O}] + n_g[\mathcal{O}] + \frac{1}{2}n_f[\mathcal{O}], \tag{3.21}$$

where $n_\partial[\mathcal{O}]$ is the number of derivatives entering the operator $\mathcal{O}$, $n_g[\mathcal{O}]$ the number of gauge coupling constants and $n_f[\mathcal{O}]$ the number of fermion fields. This infrared counting rule provides the basis for the ordering of contributions to the effective lagrangian (3.1).

B. Generalized Weinberg power-counting formula

We consider a connected Feynman diagram Γ built from vertices $\mathcal{O}_v$ of the lagrangian (3.1) labelled by $v = 1, \cdots, V$. Replacing external lines by the corresponding fields, the diagram Γ can be compared with an operator of the effective lagrangian (3.1). We call its infrared dimension $d_{\rm IR}[\Gamma]$. The low-energy limit implies a rescaling of gauge couplings and external and internal momenta

$$p_i \;\longmapsto\; tp_i, \tag{3.22}$$

$$g \;\longmapsto\; tg. \tag{3.23}$$

The masses must be rescaled as well

$$M_{W,Z} \;\longmapsto\; tM_{W,Z}, \tag{3.24}$$

$$m_\chi \;\longmapsto\; t^n m_\chi,\; n \geqslant 1, \tag{3.25}$$

and all external fermion fields

$$\chi^{\rm ext} \;\longmapsto\; t^{1/2}\chi^{\rm ext}. \tag{3.26}$$

$d_{\rm IR}[\Gamma]$ appears as the homogeneous degree of the diagram Γ in the low-energy limit $t \longrightarrow 0$

$$\Gamma \;\longmapsto\; t^{d_{\rm IR}[\Gamma]}\Gamma. \tag{3.27}$$

Hence, $d_{\rm IR}[\Gamma]$ measures the degree of suppression of a Feynman diagram in the low-energy expansion, i.e. for $t \longrightarrow 0$. This is true if one uses dimensional regularization and a mass-independent renormalization scheme, in which case there are no powers of a cut-off involved, and the naive power-counting is valid [34, 35]. As re-derived in Appendix A, $d_{\rm IR}[\Gamma]$ is given by [2, 10] (see also [36])

$$d_{\rm IR}[\Gamma] \;=\; 2 + 2L + \sum_{v=1}^{V}\left(d_{\rm IR}[\mathcal{O}_v] - 2\right), \tag{3.28}$$

where L is the number of loops and $d_{\rm IR}[\mathcal{O}_v]$ is the infrared dimension (3.21) of the vertex $\mathcal{O}_v$. This formula is formally analogous to Weinberg's power-counting formula, originally introduced in an effective theory involving GBs only [2]. The point is that it can also include vector fields (when introduced as gauge fields) and chiral fermions (if they remain massless, or if their masses can consistently be counted as $\mathcal{O}\left(p^1\right)$).

The result (3.28) calls for two comments. The first is that, for a given precision (a given chiral order $d_{\rm IR}$), only a finite number of diagrams contributes, since all terms on the right-hand side are positive: we have $L \geqslant 0$, and, if the effective lagrangian only contains terms of order $\mathcal{O}\left(p^2\right)$ or higher, $d_{\rm IR}[\mathcal{O}] \geqslant 2$. This last point is crucial to ensure that the low-energy limit is weakly interacting. In particular, in a LEET containing scalar fields other than GBs (or superpartners of chiral fermions), the requirement $d_{\rm IR}[\mathcal{O}] \geqslant 2$ is difficult to satisfy.

The second comment is that the chiral expansion is intimately related to a loop expansion: the order $d_{\rm IR}$ of a diagram increases with the number of loops L. More precision can systematically be achieved by going to the next order in the expansion, involving Feynman diagrams with one additional loop. The divergences in loops are absorbed by the (finite number of) new operators appearing at the corresponding order $d_{\rm IR}$ in the lagrangian,

yielding finite and renormalization-scale independent results [6]. See also Appendix B in this respect.

C. What becomes of the irrelevant operators of the SM in the Higgs-less effective theory?

In the simplest effective theory approach to Higgs-less EWSB, one constructs the most general lagrangian with the same fields as in Section II, except that the Higgs fields (the matrix Φ with mass-dimension one) is replaced by the 2×2 unitary and unimodular matrix Σ, describing the three GBs. The most important difference is that the rule for ordering the operators is changed from (2.8) to (3.1). On the other hand, the assumed symmetry is the same as in the SM, i.e. $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. As shown below, basing the construction on this sole symmetry leads to difficulties both in the comparison with experiment, and for the consistency of the expansion itself. This will be remedied in Section IV.

1. The $\mathcal{O}\left(p^1\right)$ lepton-number violating operator

Using a left-handed lepton doublet ℓ_L transforming as in (2.4) with $B-L=-1$, one can construct the following $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant, which breaks custodial symmetry as well as lepton number, compare (2.9)

$$\Lambda\overline{\ell_L}\Sigma\tau^+\Sigma^\dagger\left(\ell_L\right)^c \;=\; \mathcal{O}\left(p^1\right). \tag{3.29}$$

According to the power-counting rules given in III A, it appears at $\mathcal{O}\left(p^1\right)$, without any suppression factor [7]. Since this operator (3.29) has chiral dimension less than two, it violates the requirement of $d_{\mathrm{IR}}\left[\mathcal{O}\right]\geqslant 2$, which was pointed out at the end of Section III A to be crucial for the low-energy expansion to make sense. We conclude that the presence, at this order, of the lepton-number violating effective operator (3.29) would not only ruin any chance of phenomenological success of such a LEET, but also endanger its very consistency.

2. A whole class of $\mathcal{O}\left(p^2\right)$ operators

Among all $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$-invariant operators of lowest order in the Higgs-less theory, one finds $\mathcal{O}\left(p^2\right)$ operators which have no equivalent in the renormalizable lagrangian of the SM. On the other hand, one could build the corresponding invariants using the fields of the SM: they would have mass-dimension six, and their effects are therefore suppressed by a two powers of a dimensionful scale in the SM case. In absence of the Higgs particle, due the concomitant "replacement" of Φ by Σ, this suppression by a dimensionful scale no longer holds: in addition to their appearing at $\mathcal{O}\left(p^2\right)$, these operators are not dimensionally suppressed anymore. The first operator can be described as "giving a tree-level contribution to the S parameter" [6, 15]

$$b_{\mu\nu}\left\langle\Sigma\frac{\tau^3}{2}\Sigma^\dagger G^{\mu\nu}\right\rangle \;=\; \mathcal{O}\left(p^2\right), \tag{3.30}$$

and the second one as "contributing to the T parameter" [16] [8]

$$f^2\left\langle\frac{\tau^3}{2}\Sigma^\dagger D_\mu\Sigma\right\rangle^2 \;=\; \mathcal{O}\left(p^2\right). \tag{3.31}$$

If such operators appear at $\mathcal{O}\left(p^2\right)$, then they modify the two-point functions of vector fields already at that level. If they appear at tree-level, these two operators can be directly interpreted as oblique corrections: with our normalization of (3.31), the constants appearing in front of them are then, respectively identified as $-gg'S/\left(16\pi\right)$ and $e^2T/\left(64\pi\right)$. Here again, it seems that a direct application of LEETs to Higgs-less EWSB over-predicts deviations from the SM: compared to the default estimate of 1, one would need a suppression by more than a factor of 4π in order to agree with the current limits from [37, 38], of order 10^{-3}. In addition, rather than imposing a suppression by hand on these two operators, we would like it to be systematic, based on a symmetry that protects it.

Other "unwanted" operators involve fermions: the following non-universal couplings [5, 17] to massive vector bosons appear at $\mathcal{O}\left(p^2\right)$

$$\mathrm{i}\overline{\chi_L^i}\gamma^\mu\left(\Sigma D_\mu\Sigma^\dagger\right)\chi_L^j \;=\; \mathcal{O}\left(p^2\right), \tag{3.32}$$

$$\mathrm{i}\overline{\chi_R^i}\gamma^\mu\left(\Sigma^\dagger D_\mu\Sigma\right)\chi_R^j \;=\; \mathcal{O}\left(p^2\right). \tag{3.33}$$

In addition, (3.33) introduces couplings of the right-handed fermions to the $W^\pm$. Both types of operators would also be a new source of flavor-changing currents, requiring a redefinition of the CKM matrix, which would not be unitary anymore. These operators would also introduce flavor-changing neutral currents (FCNCs) at this level.

The magnetic moment operators such as

$$\frac{1}{\Lambda}\overline{\chi_L}\Sigma\sigma^{\mu\nu}\chi_R b^{\mu\nu} \;=\; \mathcal{O}\left(p^2\right), \tag{3.34}$$

$$\frac{1}{\Lambda}\overline{\chi_L}G_{\mu\nu}\Sigma\sigma^{\mu\nu}\chi_R \;=\; \mathcal{O}\left(p^2\right), \tag{3.35}$$

---

[6] This has been checked explicitly at one loop for the bosonic sector of an effective theory of EWSB without a Higgs in [12].

[7] The operator in equation (3.29) appears multiplied by the appropriate power of Λ (or f) in order to match the dimension of a term in the lagrangian. One can always argue as to which one is more appropriate, the difference being a factor of 4π. This does not upset the argument regarding the formal ordering of operators in the low-energy expansion.

[8] Once again, footnote 7 applies.

also have chiral dimension two, but mass-dimension five, and can therefore be suppressed thanks to a dimensional scale. Nonetheless, this suppression is not as strong as in the linear case: these operators would have mass-dimension six if constructed with SM fields. In fact, if the scale appearing in front of these operators was indeed $\Lambda_{\mathrm{w}} \sim 3\,\mathrm{TeV}$, the anomalous magnetic moment of the muon generated would still be larger than is measured [39, 40].

The operators (3.30-3.33) have already been mentioned in [14], but the case of the lepton-number violating one (3.29) was not fully analyzed, and the common interpretation of this class of operators as corresponding to mass-dimension six operators in the SM was not given.

There are other $\mathcal{O}\left(p^2\right)$ operators which correspond to irrelevant operators in the SM (four-fermion interactions), but they are also suppressed by a scale in the Higgs-less effective theory. Such operators are not related to symmetry breaking (they do not involve the Higgs doublet in the SM or the GB matrix in the Higgs-less case), the scale involved need not be the same as $\Lambda_{\mathrm{w}} \sim 3\,\mathrm{TeV}$, but may be larger. If so, the difficulty would not be more acute that in the SM. Therefore, we disregard these operators in this paper, since we have no control over the values of these dimensionful scales: we simply assume that they are large enough so that we can neglect the corresponding operators.

3. *Dirac mass terms*

Although they were not irrelevant in the SM, we also mention Yukawa terms, which have mass-dimension three in the Higgs-less case. One finds that they can be written down at $\mathcal{O}\left(p^1\right)$

$$\Lambda\overline{\chi_L}\Sigma\chi_R \;=\; \mathcal{O}\left(p^1\right), \tag{3.36}$$

$$\Lambda\overline{\chi_L}\Sigma\tau^3\chi_R \;=\; \mathcal{O}\left(p^1\right). \tag{3.37}$$

This is dangerous for the consistency of the expansion: the Weinberg power-counting formula (3.28) then shows that the loop expansion does not make sense, since there are operators with degree $d_{\mathrm{IR}}\left[\mathcal{O}\right] < 2$.

IV. HIGHER SYMMETRY S_{nat}, ITS REDUCTION AND SPURIONS

We ask whether an expansion procedure exists that is consistent with the principles of a LEET, and in which the unwanted operators of the previous section are relegated to higher orders. This is the motivation behind the S_{nat} symmetry and the spurion formalism.

A. The symmetry group S_{nat}

To achieve the aforementioned goal, we require the lagrangian to be invariant under a symmetry group S_{nat}, which contains as a subgroup the electroweak one $S_{\mathrm{red}} = \mathrm{SU}\left(2\right)_L \times \mathrm{U}\left(1\right)_Y$. The symmetry group S_{nat} is a product of groups acting separately on the composite sector (which produces the three GBs as its only massless bound states) and separately on the elementary sector (quarks, leptons and Yang-Mills fields). The two sectors are coupled via constraints from which the existence of spurions follows. In reference [14], the group $S_{\mathrm{nat}} = \left[\mathrm{SU}\left(2\right) \times \mathrm{SU}\left(2\right)\right]^2 \times \mathrm{U}\left(1\right)$ was shown to be large enough to introduce a suppression of the operators described in (3.30-3.33). Since we are interested in introducing the minimal number of particles, we will stick to this group. In order to clarify the discussion, the formalism is rephrased from the onset, implying some modifications with respect to the generic notation of [14].

We assume an underlying theory responsible for the spontaneous symmetry breaking of $\mathrm{SU}\left(2\right)_{\Gamma_L} \times \mathrm{SU}\left(2\right)_{\Gamma_R} \subset S_{\mathrm{nat}}$ down to its vector subgroup. This produces a triplet of GBs, which we parametrize by a unitary unimodular matrix Σ transforming according to

$$\Sigma \;\longmapsto\; \Gamma_L \Sigma \Gamma_R^\dagger, \tag{4.1}$$

where $\Gamma_L \in \mathrm{SU}\left(2\right)_{\Gamma_L}$ and $\Gamma_R \in \mathrm{SU}\left(2\right)_{\Gamma_R}$. The global $\mathrm{SU}\left(2\right)_{\Gamma_L} \times \mathrm{SU}\left(2\right)_{\Gamma_R}$ symmetry is then promoted to a local one, in order to define a generating functional for its Noether currents. The corresponding $\Gamma_{L\mu}$ and $\Gamma_{R\mu}$ sources transform according to

$$\Gamma_{L\mu} \;\longmapsto\; \Gamma_L \Gamma_{L\mu} \Gamma_L^\dagger + \mathrm{i}\Gamma_L \partial_\mu \Gamma_L^\dagger, \tag{4.2}$$

$$\Gamma_{R\mu} \;\longmapsto\; \Gamma_R \Gamma_{R\mu} \Gamma_R^\dagger + \mathrm{i}\Gamma_R \partial_\mu \Gamma_R^\dagger. \tag{4.3}$$

This composite sector will be at the origin of the electroweak symmetry breaking once gauge fields are appropriately coupled to its conserved currents. Note that the $\mathrm{SU}\left(2\right)_{\Gamma_L} \times \mathrm{SU}\left(2\right)_{\Gamma_R}$ structure parallels that of the Higgs sector in the renormalizable SM, see Section II A, and is necessary for the custodial symmetry to play its role.

Turning now to the elementary sector, we recall the formula (2.7), which indicates that we have to introduce an $\mathrm{SU}\left(2\right)_{G_L} \times \mathrm{SU}\left(2\right)_{G_R} \times \mathrm{U}\left(1\right)_{B-L}$ structure acting on elementary fermions [9]. The elementary $G_{L\mu}$ and $G_{R\mu}$ gauge fields transform under $G_L \in \mathrm{SU}\left(2\right)_{G_L}$ and $G_R \in \mathrm{SU}\left(2\right)_{G_R}$ as

$$g_L G_{L\mu} \;\longmapsto\; G_L g_L G_{L\mu} G_L^\dagger + \mathrm{i}G_L \partial_\mu G_L^\dagger, \tag{4.4}$$

$$g_R G_{R\mu} \;\longmapsto\; G_R g_R G_{R\mu} G_R^\dagger + \mathrm{i}G_R \partial_\mu G_R^\dagger. \tag{4.5}$$

The elementary fermion doublets with $B-L=-1$ will be denoted by ℓ and those with $B-L=1/3$ by q. Their

[9] This is indeed the gauge group in Higgs-less models [41], and in left-right symmetric models [18]. Here, although we will introduce in total thirteen vector fields, they will subsequently be restricted to take values in the algebra of $\mathrm{SU}\left(2\right)_L \times \mathrm{U}\left(1\right)_Y$ in Section IV B.

transformations represent an extension of those of (2.4-2.5)

$$\chi_L \longmapsto G_L \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha}\chi_L, \qquad (4.6)$$

$$\chi_R \longmapsto G_R \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha}\chi_R. \qquad (4.7)$$

The corresponding $\mathrm{U}(1)_{B-L}$ gauge field transforms as

$$g_B G_{B\mu} \longmapsto g_B G_{B\mu} - \partial_\mu \alpha, \qquad (4.8)$$

which we rewrite, embedding $\mathrm{U}(1)$ into an $\mathrm{SU}(2)$ group, as

$$g_B G_{B\mu}\frac{\tau^3}{2} \longmapsto \mathrm{e}^{-\mathrm{i}\alpha\frac{\tau^3}{2}} g_B G_{B\mu}\frac{\tau^3}{2}\mathrm{e}^{\mathrm{i}\alpha\frac{\tau^3}{2}} + \mathrm{i}\mathrm{e}^{-\mathrm{i}\alpha\frac{\tau^3}{2}}\partial_\mu \mathrm{e}^{\mathrm{i}\alpha\frac{\tau^3}{2}}. \qquad (4.9)$$

Note that, in this formalism, the χ_R are $\mathrm{SU}(2)_{G_R}$ doublets. This implies, in particular, that we have introduced ν_R degrees of freedom.

The symmetry group of the theory is

$$S_{\mathrm{nat}} = \mathrm{SU}(2)_{G_L} \times \mathrm{SU}(2)_{\Gamma_L} \times \mathrm{SU}(2)_{\Gamma_R} \times \mathrm{SU}(2)_{G_R} \times \mathrm{U}(1)_{B-L}, \qquad (4.10)$$

hence the effective lagrangian should contain all invariants under this group, organized according to their chiral dimensions d_{IR}. The operators with chiral dimension $\mathcal{O}(p^2)$ involving the fields introduced at this stage are collected in $\mathcal{L}(p^2)$

$$\mathcal{L}(p^2) = \frac{f^2}{4}\langle D_\mu \Sigma^\dagger D^\mu \Sigma\rangle + \mathrm{i}\overline{\chi_L}\gamma^\mu D_\mu \chi_L + \mathrm{i}\overline{\chi_R}\gamma^\mu D_\mu \chi_R - \frac{1}{2}\langle G_{L\mu\nu}G_L^{\mu\nu} + G_{R\mu\nu}G_R^{\mu\nu}\rangle - \frac{1}{4}G_{B\mu\nu}G_B^{\mu\nu}. \qquad (4.11)$$

The covariant derivatives are defined in the standard manner in terms of the connections $\Gamma_{L\mu}, \Gamma_{R\mu}, g_L G_{L\mu}, g_R G_{R\mu}, g_B G_{B\mu}$, following the transformation properties (4.1) and (4.6-4.7).

Equation (4.11) contains all S_{nat}-invariant operators that are $\mathcal{O}(p^2)$ except four-fermion operators without derivatives. The latter are dimensionally suppressed by the inverse squared of a scale $\Lambda_{4\mathrm{f}}$ which we assume to be much higher than the scale Λ_{w} (1.1) of the LEET. Notice that such $\mathcal{O}(p^2)$ four-fermion operators cannot be generated by loops.

B. Covariant reduction of the symmetry $S_{\mathrm{nat}} \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

The symmetry S_{nat} is large enough to eliminate all the unwanted couplings discussed in Section III C at the leading chiral order $\mathcal{O}(p^2)$ described by the lagrangian (4.11). On the other hand, S_{nat} is too large: the lagrangian (4.11) contains thirteen gauge connections $g_L G_{L\mu}, g_R G_{R\mu}, g_B G_{B\mu}, \Gamma_{L\mu}, \Gamma_{R\mu}$, as compared to four in the SM. Due to the GB term in (4.11) (first term on the right-hand side), the three combinations $\Gamma^a_{R\mu} - \Gamma^a_{L\mu}$ acquire a mass term by the Higgs mechanism, whereas all ten remaining vector fields as well as fermions remain massless. Furthermore, the lagrangian (4.11) does not contain any coupling that would transmit the symmetry breaking from the composite sector $(\Sigma, \Gamma_{L\mu}, \Gamma_{R\mu})$ to the elementary sector $(G_{L\mu}, G_{R\mu}, G_{B\mu}, \chi)$. Following [14], such a coupling will be introduced —and the number of degrees of freedom reduced— via constraints identifying certain connections of S_{nat} up to a gauge transformation. This will provide the reduction $S_{\mathrm{nat}} \longrightarrow \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. In order not to loose the benefit of the large symmetry S_{nat}, the constraints must be invariant under S_{nat}. We show that such a procedure is tantamount to introducing a set of unitary fields with definite transformations under S_{nat} and vanishing covariant derivatives. Introducing multiplication factors into these matrices, one obtains the spurions of [14]. The reciprocal —i.e. postulating the existence of covariantly constant spurions and thereby obtaining the reduction— was already discussed in [14, 20].

1. *Origin of spurions*

We want to identify $\Gamma_{L\mu}$ to $g_L G_{L\mu}$, up to a gauge transformation $\Omega_L \in \mathrm{SU}(2)$, i.e.

$$\Gamma_{L\mu} = \Omega_L(x)\, g_L G_{L\mu}\Omega_L^{-1}(x) + \mathrm{i}\Omega_L(x)\,\partial_\mu \Omega_L^{-1}(x). \qquad (4.12)$$

This will reduce the group $\mathrm{SU}(2)_{G_L} \times \mathrm{SU}(2)_{\Gamma_L}$ to its vector subgroup, which will be recognized as the $\mathrm{SU}(2)_L$ introduced in the SM —see Section II A. Requiring the invariance of the constraint (4.12) with respect to the whole symmetry $\mathrm{SU}(2)_{G_L} \times \mathrm{SU}(2)_{\Gamma_L}$ amounts to promoting the gauge function Ω_L to a field variable transforming according to

$$\Omega_L \longmapsto \Gamma_L \Omega_L G_L^\dagger. \qquad (4.13)$$

Note that equation (4.12) now replaces the original set of gauge connections $\{G_{L\mu}, \Gamma_{L\mu}\}$ by a smaller set $G_{L\mu}$ *and* Ω_L, maintaining a well-defined action of the $\mathrm{SU}(2)_{G_L} \times \mathrm{SU}(2)_{\Gamma_L}$ symmetry group on this smaller manifold $\{G_{L\mu}, \Omega_L\}$. The field Ω_L is *not* a GB, but rather a non-propagating *spurion*: this follows from the constraint (4.12) itself, since the latter can be equivalently rewritten as the condition of covariant constancy of Ω_L, reflecting the transformation (4.13)

$$D_\mu \Omega_L \equiv \partial_\mu \Omega_L - \mathrm{i}\Gamma_{L\mu}\Omega_L + \mathrm{i}g_L \Omega_L G_{L\mu} = 0. \qquad (4.14)$$

Next, we proceed to a similar reduction in the right-handed sector. The connections $\Gamma_{R\mu}$ and $G_{R\mu}$ are identified up to a gauge through the constraint

$$\Gamma_{R\mu} = \Omega_R(x)\, g_R G_{R\mu}\Omega_R^{-1}(x) + \mathrm{i}\Omega_R(x)\,\partial_\mu \Omega_R^{-1}(x). \qquad (4.15)$$

This is done by introducing of the spurion $\Omega_R \in \mathrm{SU}(2)$ transforming as

$$\Omega_R \ \longmapsto \ \Gamma_R \Omega_R G_R^\dagger, \tag{4.16}$$

and enforcing the covariant constancy of Ω_R

$$D_\mu \Omega_R \ = \ \partial_\mu \Omega_R - \mathrm{i}\Gamma_{R\mu}\Omega_R + \mathrm{i}g_R \Omega_R G_{R\mu} \ = \ 0 \tag{4.17}$$

The action of the whole symmetry $\mathrm{SU}(2)_{\Gamma_R} \times \mathrm{SU}(2)_{G_R}$ on the reduced manifold $\{G_{R\mu}, \Omega_R\}$ is still at work, but only the diagonal subgroup —identified with the right-handed isospin $\mathrm{SU}(2)_R$ — is linearly realized in the space of propagating fields.

So far, the symmetry S_{nat} has been reduced to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, involving seven gauge connections and two spurions fields Ω_L, Ω_R. In order to end up with the physical degrees of freedom of the electroweak sector, with $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as the maximal linearly realized symmetry, it remains to further reduce $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ to $\mathrm{U}(1)_Y$. This can be achieved identifying $\mathrm{SU}(2)_R$ with $\mathrm{U}(1)_{B-L}$ embedded into $\mathrm{SU}(2)$ according to equation (4.9). Such an identification amounts to the constraint

$$\Gamma_{R\mu} \ = \ \omega_\Gamma g_B G_{B\mu} \frac{\tau^3}{2} \omega_\Gamma^{-1} + \mathrm{i}\omega_\Gamma \partial_\mu \omega_\Gamma^{-1}, \tag{4.18}$$

where the spurion $\omega_\Gamma \in \mathrm{SU}(2)$ transforms as

$$\omega_\Gamma \ \longmapsto \ \Gamma_R \omega_\Gamma \mathrm{e}^{\mathrm{i}\alpha \frac{\tau^3}{2}}. \tag{4.19}$$

Again, the constraint (4.18) is equivalent to the vanishing of the corresponding covariant derivative

$$D_\mu \omega_\Gamma \ \equiv \ \partial_\mu \omega_\Gamma - \mathrm{i}\Gamma_{R\mu}\omega_\Gamma + \mathrm{i}g_B \omega_\Gamma G_{B\mu} \frac{\tau^3}{2} \ = \ 0 \tag{4.20}$$

We note in passing, that the constraints (4.15) and (4.18) imply, by transitivity, the relation

$$g_R G_{R\mu} \ = \ \omega_G g_B G_{B\mu} \frac{\tau^3}{2} \omega_G^{-1} + \mathrm{i}\omega_G \partial_\mu \omega_G^{-1}, \tag{4.21}$$

where ω_G is defined as

$$\omega_G \ \equiv \ \Omega_R^\dagger \omega_\Gamma \ \longmapsto \ G_R \omega_\Gamma \mathrm{e}^{\mathrm{i}\alpha \frac{\tau^3}{2}}. \tag{4.22}$$

The constraint (4.18) implies the orientation of the right-isospin in the third direction together with the reduction of the remaining symmetry $\mathrm{U}(1)_{T_R^3} \times \mathrm{U}(1)_{B-L}$ to its diagonal subgroup $\mathrm{U}(1)_Y$ where

$$\frac{Y}{2} \ = \ T_R^3 + \frac{B-L}{2}. \tag{4.23}$$

We thus end up with the reduced symmetry

$$S_{\mathrm{red}} \ = \ \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \,. \tag{4.24}$$

The reduction of the symmetry from S_{nat} to S_{red} is schematically represented in FIG. 1. The symmetry S_{red}

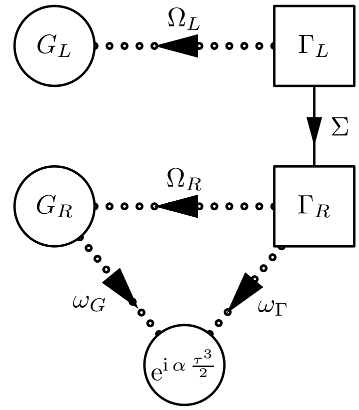

FIG. 1: Transformation properties of *unitary* spurions (dotted lines) and GBs (continuous line).

(4.24) acts linearly on physical fields, exactly as in the SM. In particular, the right-handed neutrino is inert under S_{red} (4.24) and in the lagrangian (4.11), it decouples once the constraints (4.12), (4.15) and (4.18) are applied. Yet, its presence in the theory is enforced by the symmetry S_{nat}. The latter did not disappear: the transformations belonging to

$$S_{\mathrm{hidden}} \ = \ \frac{S_{\mathrm{nat}}}{S_{\mathrm{red}}}, \tag{4.25}$$

merely act on the spurion fields $\Omega_L, \Omega_R, \omega_G$ and consequently they become apparent only when terms explicitly involving spurions are included.

2. Magnitude of spurions

The covariant constraints (4.14), (4.20) and (4.21) can now be plugged into the $\mathcal{O}\left(p^2\right)$ lagrangian (4.11), eliminating nine of the connections in favor of four $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge fields, and the unitary fields ω_Γ, ω_G and Ω_L. The latter keep track of the original symmetry S_{nat}, they do not propagate (their covariant derivatives vanish, c.f. (4.14), (4.20) and (4.21)) and they can be transformed away by redefinition of the remaining fields. The lagrangian then involves the four $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge fields with all vertices, mixing and vector boson masses identical to the SM without the following: left-over Higgs particle, Yukawa couplings and fermion mass terms.

The problem of unwanted terms reappears as long as the spurions are described by the unitary variables $\omega_G, \omega_\Gamma, \Omega_L$, introducing no new parameters that cannot be eliminated by a gauge choice: the spurions $\omega_G, \omega_\Gamma, \Omega_L$ can be used to construct other S_{nat}-invariants that are still $\mathcal{O}\left(p^2\right)$ but are not contained in (4.11). These additional terms are exactly all the terms of the type (3.29-3.35), which still have to be suppressed.

This can be achieved by adding a new ingredient: we now admit multiplication of the unitary spurions by small constants (expansion parameters), which does not modify

(4.12) and (4.15). For the first identifications (4.14) and (4.17), this is implemented via the requirement that only the objects $\mathcal{X},\mathcal{Y}$

$$\mathcal{X} \equiv \xi\Omega_L, \quad (4.26)$$
$$\mathcal{Y} \equiv \eta\Omega_R, \quad (4.27)$$

and positive powers of them or of their hermitian conjugate may be inserted in the operators of Section III C in order to make them invariant under $S_{\rm nat}$. The order of magnitude of ξ and η should later be estimated from experiments, but they will be considered as expansion parameters. Indeed, the $S_{\rm nat}$ symmetry guarantees that the condition of small ξ,η will not be upset by the loop expansion.

It is convenient to decompose $\mathcal{Y}$ as

$$\mathcal{Y} = \mathcal{Y}_u + \mathcal{Y}_d, \quad (4.28)$$

where

$$\mathcal{Y}_u \equiv \eta\omega_\Gamma \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \omega_G^\dagger \longmapsto \Gamma_R \mathcal{Y}_u G_R^\dagger, \quad (4.29)$$
$$\mathcal{Y}_d \equiv \mathcal{Y}_{uc} \equiv \tau^2 \mathcal{Y}_u^* \tau^2 = \eta\omega_\Gamma \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \omega_G^\dagger \longmapsto \Gamma_R \mathcal{Y}_d G_R^\dagger. \quad (4.30)$$

Note also the following relations, useful for the future construction of the lagrangian

$$\mathcal{Y}_u \mathcal{Y}_d^\dagger = 0, \quad (4.31)$$
$$\mathcal{Y}_u \mathcal{Y}_u^\dagger + \mathcal{Y}_d \mathcal{Y}_d^\dagger = \eta^2 \mathbb{1}. \quad (4.32)$$

As for $\mathcal{X}$, its transformation law is immediate from the definition (4.26)

$$\mathcal{X} \longmapsto \Gamma_L \mathcal{X} G_L^\dagger. \quad (4.33)$$

For the triangular identification (4.15), (4.18) and (4.21) in the right-handed sector, one must specify two independent combinations of unitary matrices Ω_R , ω_Γ and ω_G in front of which small expansion parameters will be introduced. We have already performed a choice when writing (4.27) and/or (4.29-4.30). This choice is motivated by the fact that $\mathcal{Y}_u$ and $\mathcal{Y}_d$ are neutral under $\mathrm{U}(1)_{B-L}$, whereas both ω_Γ and ω_G carry a non-zero $B-L$ charge. Furthermore, it is an empirical fact that the right-handed isospin violation is less suppressed than the violation of $B-L$. It remains to specify how the remaining $\mathrm{U}(1)_{B-L}$ breaking spurion strength will be introduced. For the moment, we focus on the simplest case, which is most symmetric between the left- and right-handed sectors —but is not the one presented in [14]. The general case will be discussed in section IV B 4. We define

$$\mathcal{Z} \equiv \zeta_{\rm I}^2 \omega_\Gamma \tau^+ \omega_\Gamma^\dagger. \quad (4.34)$$

From (4.19), we find that the spurion $\mathcal{Z}$ so defined transform as

$$\mathcal{Z} \longmapsto \mathrm{e}^{+\mathrm{i}\alpha} \Gamma_R \mathcal{Z} \Gamma_R^\dagger. \quad (4.35)$$

All the other objects that can be constructed from products of $\mathcal{X},\mathcal{Y}_{u,d}$ and $\mathcal{Z}$ automatically carry definite powers of ξ,η or $\zeta_{\rm I}$. Note that $\mathcal{Z}$ and $\mathcal{Z}^\dagger$satisfy a Grassman algebra

$$\left\{\mathcal{Z},\mathcal{Z}^\dagger\right\} = \zeta_{\rm I}^4 \mathbb{1}, \quad (4.36)$$
$$\mathcal{Z}^2 = 0. \quad (4.37)$$

Other related properties of the spurions that can be deduced from the above are

$$\mathcal{Y}_d^\dagger \mathcal{Z} = \mathcal{Z}\mathcal{Y}_u = 0, \quad (4.38)$$
$$\mathcal{Z}^\dagger = -\mathcal{Z}_c, \quad (4.39)$$
$$\mathrm{Tr}\,\mathcal{Z} = 0. \quad (4.40)$$

3. *Reciprocal*

The reciprocal of the statement (Section IV B 1) that spurions are a necessary consequence of the covariant identification of subgroups of $S_{\rm nat}$ also holds. To show this, one considers a set of three spurions $\mathcal{X},\mathcal{Y}_u$ and $\mathcal{Z}$ transforming as in (4.33), (4.29) and (4.35) and satisfying (4.31) and (4.37), and imposes the following constraints on them

$$D_\mu \mathcal{X} \equiv \partial_\mu \mathcal{X} - \mathrm{i}\Gamma_{L\mu}\mathcal{X} + \mathrm{i}g_L \mathcal{X} G_{L\mu} = 0, \quad (4.41)$$
$$D_\mu \mathcal{Y}_u \equiv \partial_\mu \mathcal{Y}_u - \mathrm{i}\Gamma_{R\mu}\mathcal{Y}_u + \mathrm{i}g_R \mathcal{Y}_u G_{R\mu} = 0, \quad (4.42)$$
$$D_\mu \mathcal{Z} \equiv \partial_\mu \mathcal{Z} - \mathrm{i}g_R \left[\Gamma_{R\mu},\mathcal{Z}\right] + \mathrm{i}g_B G_{B\mu}\mathcal{Z} = 0 \quad (4.43)$$

One then finds that equations (4.41-4.43) have no nontrivial solution ($\mathcal{X},\mathcal{Y}_u,\mathcal{Z} \neq 0$) unless the thirteen gauge connections of $S_{\rm nat}$ are aligned in a definite way. As shown in [14], the integrability condition of the system (4.41-4.43) amounts to the existence of a gauge, specified by $\Omega_L = \omega_G = \omega_\Gamma = 1$. In this gauge, called the "standard gauge", we automatically have $\Omega_R \equiv \omega_G \omega_\Gamma^\dagger = 1$ and

$$\Gamma_{L\mu} \overset{\rm s.g.}{=} g_L G_{L\mu}, \quad (4.44)$$
$$\Gamma_{R\mu}^{1,2} \overset{\rm s.g.}{=} g_R G_{R\mu}^{1,2} \overset{\rm s.g.}{=} 0, \quad (4.45)$$
$$\Gamma_{R\mu}^{3} \overset{\rm s.g.}{=} g_R G_{R\mu}^{3} \overset{\rm s.g.}{=} g_B G_{B\mu}^{3}. \quad (4.46)$$

In the aforementioned gauge, the spurions reduce to three real functions $\xi(x)$, $\eta(x)$ and $\zeta_{\rm I}(x)$ according to

$$\mathcal{X} \overset{\rm s.g.}{=} \xi \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (4.47)$$
$$\mathcal{Y}_u \overset{\rm s.g.}{=} \eta \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathcal{Y}_d \overset{\rm s.g.}{=} \eta \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (4.48)$$
$$\mathcal{Z} \overset{\rm s.g.}{=} \zeta_{\rm I}^2 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (4.49)$$

The constraints (4.41-4.43) also yield

$$\partial_\mu \xi = \partial_\mu \eta = \partial_\mu \zeta_{\rm I} = 0, \quad (4.50)$$

stating the space-time independence of ξ, η and $\zeta_{\rm I}$. This is the reciprocal of Section IV B 1.

The solution (4.47-4.49) and the conditions (4.44-4.46) are invariant under the electroweak group, which is a subgroup of $S_{\rm nat}$

$$S_{\rm red} \;=\; {\rm SU}\left(2\right)_L \times {\rm U}\left(1\right)_Y . \tag{4.51}$$

$S_{\rm red}$ involves the four SM gauge fields G^a_μ and b_μ defined here through the values of the other fields in the gauge where (4.44-4.46) hold

$$gG^a_\mu \;\equiv\; g_L G^a_{L\mu}|_{\rm s.g.} \left(= \Gamma^a_{L\mu}|_{\rm s.g.}\right), \tag{4.52}$$

$$g'b_\mu \;\equiv\; g_R G^3_{R\mu}|_{\rm s.g.} \left(= g_B G_{B\mu}|_{\rm s.g.} = \Gamma^3_{R\mu}|_{\rm s.g.}\right) \tag{4.53}$$

The kinetic ${\rm U}\left(1\right)_Y$ term is normalized if g' is defined as

$$\frac{1}{g'^2} \;\equiv\; \frac{1}{g_R^2} + \frac{1}{g_B^2}. \tag{4.54}$$

It has been shown [14] that, upon inserting the constraints (4.41-4.43) into the leading-order lagrangian (4.11), the SM couplings are obtained, between massive $W^\pm$, Z^0 and photon and *massless* fermions. Following the Higgs mechanism, the three GBs contained in the matrix Σ are absorbed by the $W^\pm$ and Z^0, and there is no scalar particle left in the spectrum. The definitions of the gauge fields diagonalizing the quadratic terms in the lagrangian are

$$W^\pm_\mu \;\equiv\; \frac{{\rm i}\sqrt{2}}{g} \left\langle \tau^\mp \Sigma^\dagger D_\mu \Sigma \right\rangle|_{\rm s.g.}, \tag{4.55}$$

$$Z_\mu \;\equiv\; \frac{{\rm i}c}{g} \left\langle \tau^3 \Sigma^\dagger D_\mu \Sigma \right\rangle|_{\rm s.g.}, \tag{4.56}$$

$$A_\mu \;\equiv\; \frac{{\rm i}s}{g} \left\langle \tau^3 \Sigma^\dagger D_\mu \Sigma \right\rangle|_{\rm s.g.} + \frac{1}{c} G_{B\mu}|_{\rm s.g.}, \tag{4.57}$$

where s and c are the sinus and cosinus of the Weinberg angle $s = \frac{g'}{\sqrt{g^2+g'^2}}$, $c = \frac{g}{\sqrt{g^2+g'^2}}$.

In all of the sequel, the constants ξ, η and $\zeta_{\rm I}$ will be treated as small expansion parameters. This is consistent since, in the absence of spurions, one recovers the larger symmetry $S_{\rm nat}$. This is one role of the spurions: to introduce in a covariant manner these tunable parameters. Covariance is necessary in order to classify various operators according to their symmetry-breaking properties. The other role of spurions is to select the vacuum alignment, i.e. the embedding of the electroweak group $S_{\rm red}$ in $S_{\rm nat}$: this is the identification of connections given in (4.44-4.46).

The next step in the formulation of the LEET is the construction of the effective lagrangian: one writes down all terms invariant under $S_{\rm nat}$ that can be constructed out of the GBs Σ, the connections $\Gamma_{L\mu}, \Gamma_{R\mu}$, the gauge fields $G_{L\mu}, G_{R\mu}, G_{B\mu}$, the spurions $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$, and fermions. The operators should be ordered according to their chiral power-counting and, in addition, according to the powers of spurions involved. To exhibit the physical content of each operator, one then injects the solution (4.44-4.50) of the constraints. This yields a lagrangian depending on the fermions and on the $S_{\rm red}$ gauge fields G_μ and b_μ, which should be used as dynamical variables to compute loops. In addition, this lagrangian depends on the three constants ξ, η and $\zeta_{\rm I}$. At the leading order $\mathcal{O}\left(p^2\right)$ without explicit powers of spurions, the lagrangian describes exactly the SM couplings but without the Higgs boson, and with all fermions left massless. The origin of fermion masses, which will come with explicit powers of spurions, thus appears different from that of vector bosons. Other terms involving explicit powers of spurions will also bring other interactions. This is the subject of Sections V, VI and VII.

4. Magnitude of $(B-L)$-breaking

Before we describe remaining possibilities of introducing the $(B-L)$-breaking spurion strength and discuss their physical content, it is worth stressing that the number of spurions to be introduced is entirely fixed once we have identified the higher $S_{\rm nat}$ symmetry, and once we ask to recover the electroweak group $S_{\rm red}$ imposing constraints. As should be clear from the discussion of Section (IV B 1), the introduction of spurions is not a choice, but follows from the requirement that the formalism be covariant under $S_{\rm nat}$.

Given $S_{\rm nat} = \left[{\rm SU}\left(2\right) \times {\rm SU}\left(2\right)\right]^2 \times {\rm U}\left(1\right)_{B-L}$, exactly three expansion parameters can be introduced, that suppress the unwanted operators mentioned in Section III C. Of these three, one $(\mathcal{X})$ pertains to the left-handed sector: there is no choice up to here. In the right-handed sector, due to the triangular identification of connections (see FIG. 1), there would a priori be various possibilities for the introduction of the expansion parameters. In order that $(B-L)$-breaking remains a small effect, we required that one of the expansion parameters $(\mathcal{Y})$ does not break $(B-L)$. Otherwise, non-$(B-L)$-breaking effects would appear only at quadratic level in the $(B-L)$-breaking parameter. Once this physical motivation is spelled out, we see that there was no choice at this stage either.

It turns out that we remain with three inequivalent possibilities for the $(B-L)$-breaking building block $\mathcal{Z}$, i.e. the one that carries a non-zero $(B-L)$ charge. They correspond to the possibility of introducing the expansion parameters ζ in factor of different combinations of the unitary spurions ω_G, ω_Γ. Other cases with fundamental building blocks bilinear in ω_G, ω_Γ, can always be brought back to one of these three. We already mentioned the simplest case in Section IV B 2. Instead of $\mathcal{Z}$, we could have introduced the spurion $\mathcal{Z}_{\rm II}$

$$\mathcal{Z}_{\rm II} \;\equiv\; \zeta^2_{\rm II} \omega_\Gamma \tau^+ \omega^\dagger_G \longmapsto {\rm e}^{{\rm i}\alpha} \Gamma_R \mathcal{Z}_{\rm II} G^\dagger_R, \tag{4.58}$$

to be taken as a fundamental building block rather than $\mathcal{Z}$ —but still keeping $\mathcal{Y}$ as an elementary spurion. Another independent case is with the spurion $\mathcal{Z}_{\rm III}$ instead

of $\mathcal{Z}$

$$\mathcal{Z}_{\mathrm{III}} \equiv \zeta_{\mathrm{III}}^2 \omega_G \tau^+ \omega_G^\dagger \longmapsto \mathrm{e}^{\mathrm{i}\alpha} G_R \mathcal{Z}_{\mathrm{III}} G_R^\dagger. \quad (4.59)$$

In the three cases, all other combinations must then be constructed out of a basis of two spurions, using hermitian conjugation, but without using the inverse of a matrix.

The cases II and III could be recast as the one of Section IV B 2 if one could freely use the following rewritings

$$\mathcal{Z} \equiv \frac{\zeta_{\mathrm{I}}^2}{\zeta_{\mathrm{II}}^2 \eta} \mathcal{Z}_{\mathrm{II}} \mathcal{Y}_d^\dagger, \quad (4.60)$$

$$\mathcal{Z} \equiv \frac{\zeta_{\mathrm{I}}^2}{\zeta_{\mathrm{III}}^2 \eta^2} \mathcal{Y}_u \mathcal{Z}_{\mathrm{III}} \mathcal{Y}_d^\dagger, \quad (4.61)$$

as can be deduced from a comparison of the definition (4.34) with those of (4.58-4.59). However, the relations (4.60-4.61) are singular in the limit of vanishing spurions. Note that the possibility to build operators that produce an S_{nat}-invariant operator with a given physical content —i.e. are equal in the standard unitary gauge— does not depend on the assumed case I, II or III, but only on the symmetries. What is modified is the lowest number of spurion powers necessary to construct such an operator. The three situations labelled by I, II, III are therefore not equivalent from the point of view of magnitude estimates, and of the expansion. As we shall see in Section V C 3, a difference appears in the ratio of left- to right-handed neutrino masses. This distinction in turn implies different cosmological consequences which we explore in Section VIII, where we will even see that the physical hypothesis labeled by III seems a priori excluded.

We can now show that the case mentioned in [14] corresponds to the physical situation denoted here by III. We first note that the definition of the spurion ϕ used in [14] would in the present formalism require the introduction of the following complex doublet

$$\phi_G \equiv \omega_G \begin{pmatrix} 1 \\ 0 \end{pmatrix} \longmapsto G_R \mathrm{e}^{\mathrm{i}\frac{\alpha}{2}} \phi_G, \quad (4.62)$$

and its conjugate

$$(\phi_G)_c \equiv \mathrm{i}\tau^2 \phi_G^* \longmapsto G_R \mathrm{e}^{-\mathrm{i}\frac{\alpha}{2}} (\phi_G)_c. \quad (4.63)$$

With ϕ_G, one can construct the spurion ϕ of [14]

$$\phi \equiv \zeta_{\mathrm{III}} \phi_G. \quad (4.64)$$

providing the connection between the writing used in [14] and the present paper through the further equality

$$\mathcal{Z}_{\mathrm{III}} = \phi \phi_c^\dagger. \quad (4.65)$$

This also explains the normalization between $\mathcal{Z}_{\mathrm{III}}$ and ζ_{III} —which corresponds to the ζ of [14].

C. Accidental flavor symmetry

In the absence of spurions, the large symmetry S_{nat} forbids all non-standard couplings of the type mentioned in Section III C 2. Consequently, at the leading order, the couplings of all fermion doublets (with a given $B-L$) are identical. Furthermore, S_{nat} forbids both Dirac and Majorana mass terms [10].

If each quark and lepton doublet appears in three copies (generations or families), the lagrangian at the leading order $\mathcal{O}(p^2)$ without explicit powers of spurions necessarily enjoys a global flavor (chiral) symmetry

$$S_{\mathrm{flavor}} = \mathrm{U}(3)_L \times \mathrm{U}(3)_R, \quad (4.66)$$

separately for quarks and for leptons. Denoting by $\chi_{L,R}^i$, $i=1,2,3$ a generic left (right)-handed fermion doublet, the generators of S_{flavor} are simply

$$Q_{L,R}^{ij} = \int \mathrm{d}^3\boldsymbol{x} \overline{\chi_{L,R}^i} \gamma^0 \chi_{L,R}^j, \qquad i,j = 1,2,3 \quad (4.67)$$

Classically, they are invariant under all transformations in S_{nat}, and, at the leading order, they are conserved. At the quantum level, there will be anomalies. The symmetry S_{flavor} is accidental, since its presence at this level merely follows from the local symmetry S_{nat} and from the particle content.

The spurion $\mathcal{Y}$ allows one to extend the flavor symmetry $\mathrm{U}(3)_R$ that acts on the whole doublet χ_R^i into two independent transformations of its up and down components. Indeed, the charges Q_{Ru}^{ij} and Q_{Rd}^{ij} defined though

$$\eta^2 Q_{Ru}^{ij} = \int \mathrm{d}^3\boldsymbol{x} \overline{\chi_R^i} \gamma^0 \mathcal{Y}_u^\dagger \mathcal{Y}_u \chi_R^j, \quad (4.68)$$

$$\eta^2 Q_{Rd}^{ij} = \int \mathrm{d}^3\boldsymbol{x} \overline{\chi_R^i} \gamma^0 \mathcal{Y}_d^\dagger \mathcal{Y}_d \chi_R^j, \quad (4.69)$$

are (classically) invariant with respect to S_{nat}. They obey

$$Q_{Ru}^{ij} + Q_{Rd}^{ij} = Q_R^{ij}, \quad (4.70)$$

and commute with each other

$$[Q_{Ru}, Q_{Rd}] = 0. \quad (4.71)$$

At the leading order $\mathcal{O}(p^2)$ without explicit powers of spurions, both Q_{Ru} and Q_{Rd} are conserved, provided the constraints (4.41-4.43) hold. This is in direct connection with the absence at this order of right-handed charged current interaction. We thus have an extended flavor horizontal symmetry

$$S_{\mathrm{flavor}}^{\mathrm{extended}} = \mathrm{U}(3)_L \times \left[\mathrm{U}(3)_R^u \times \mathrm{U}(3)_R^d\right]. \quad (4.72)$$

[10] Recall that, in Sections III C 3 and III C 1, Dirac and Majorana mass terms were identified as a source of possible difficulties for the LEET.

In the absence of fermion masses, the same extended symmetry also exists in the SM, reflecting the singlet character of right-handed fermions.

A comment about the left-handed part of S_{flavor} is in order. Adding two more powers of the spurion $\mathcal{X}$, it is again possible to split $\mathrm{U}(3)_L$ into separate $\mathrm{U}(3)$ transformations of up and down components of χ_L^i. The corresponding charges invariant with respect to S_{nat} read

$$\xi^2\eta^2 Q_{Lu}^{ij} = \int \mathrm{d}^3\boldsymbol{x}\overline{\chi_L^i}\gamma^0\mathcal{X}^\dagger\Sigma\mathcal{Y}_u\mathcal{Y}_u^\dagger\Sigma^\dagger\mathcal{X}\chi_L^j, \quad (4.73)$$

$$\xi^2\eta^2 Q_{Ld}^{ij} = \int \mathrm{d}^3\boldsymbol{x}\overline{\chi_L^i}\gamma^0\mathcal{X}^\dagger\Sigma\mathcal{Y}_d\mathcal{Y}_d^\dagger\Sigma^\dagger\mathcal{X}\chi_L^j. \quad (4.74)$$

However, unlike in the right-handed case, the charges Q_{Lu} and Q_{Ld} are not separately conserved. The obstruction is due to the coupling between u_L and d_L via charged left-handed current interactions.

What was shown in this section, independently of the flavor symmetry, is that it is always possible to perform separate $\mathrm{U}(3)$ rotations of the u and d components of the doublet, in a way which is covariant under the whole symmetry S_{nat}. Such rotations are generated by the charges (4.67-4.69), no matter whether the latter are conserved or not.

V. LEADING CONTRIBUTIONS TO FERMION MASSES

In the following three sections, we consider the main spurion effects associated with leading chiral orders. Dirac masses and $(B-L)$-conserving non-standard vertices merely concern the spurions ξ and η. On the other hand, Majorana masses and $\Delta L = 2$ lepton couplings are intimately related with the spurion ζ. In general, terms in the lagrangian that contain spurions also break the horizontal flavor symmetry $S_{\text{flavor}}^{\text{extended}}$ (4.72) introduced in Section IV C. It is conceivable that different powers of spurions contributing to the same quantity could describe a hierarchical structure of $S_{\text{flavor}}^{\text{extended}}$ symmetry breaking. Hereafter we merely concentrate on the *complete list* of leading spurion effects, postponing a more detailed discussion of flavor symmetry breaking to a later stage.

A. Quark masses

Let us stress once more that the standard Yukawa couplings (3.36-3.37) are now forbidden, since they are not invariant under S_{nat}: the (composite) GBs Σ and the (elementary) fermions $\chi_{L,R}$ transform under different groups —cf. equations (4.1) and (4.6-4.7)—, reflecting their different physical origin. The lowest order Dirac mass requires one insertion of the spurion $\mathcal{X}$ and one of the spurion $\mathcal{Y}$ —either $\mathcal{Y}_u$ or $\mathcal{Y}_d$. Hence, the leading quark mass term in the lagrangian is of order $\mathcal{O}\left(p^1\xi^1\eta^1\right)$ and reads

$$\begin{aligned}\mathcal{L}\left(p^1\xi^1\eta^1\right)_{\text{quarks}} &= -\mu_{ij}^u\overline{q_L^i}\mathcal{X}^\dagger\Sigma\mathcal{Y}_u q_R^j \\ &\quad -\mu_{ij}^d\overline{q_L^i}\mathcal{X}^\dagger\Sigma\mathcal{Y}_d q_R^j + \text{h.c}, \quad (5.1)\end{aligned}$$

where $q_{L,R}^i$ denotes quark doublets. We next define doublets $q_{L,R}^{u,d}$ transforming under $\mathrm{SU}(2)_{\Gamma_R}$ through

$$\eta q_R^u \equiv \mathcal{Y}_u q_R, \quad (5.2)$$

$$\eta q_R^d \equiv \mathcal{Y}_d q_R, \quad (5.3)$$

$$\xi\left(\mathcal{Y}_u\mathcal{Y}_u^\dagger q_L^u + \mathcal{Y}_d\mathcal{Y}_d^\dagger q_L^d\right) \equiv \eta^2\Sigma^\dagger\mathcal{X}q_L, \quad (5.4)$$

and drop the generation indices on the fields u, d, since we will always collect the three families into one column. The newly-defined fields take the following form in the standard gauge

$$q_{L,R}^u \stackrel{\text{s.g.}}{=} \begin{pmatrix} u_{L,R} \\ 0 \end{pmatrix}, \quad (5.5)$$

$$q_{L,R}^d \stackrel{\text{s.g.}}{=} \begin{pmatrix} 0 \\ d_{L,R} \end{pmatrix}. \quad (5.6)$$

With the definition of two three-by-three matrices M_u and M_d through

$$(M_{u,d})_{ij} \equiv \eta\xi\mu_{ij}^{u,d}, \quad (5.7)$$

and using the standard gauge, (5.1) assumes the form

$$\begin{aligned}\mathcal{L}\left(p^1\xi^1\eta^1\right)_{\text{quarks}} &\stackrel{\text{s.g.}}{=} -\overline{u_L}M_u u_R \\ &\quad -\overline{d_L}M_d d_R + \text{h.c.} \quad (5.8)\end{aligned}$$

This shows that, at this level, the freedom for the mass matrix is the same as in the SM. Therefore, using unitary transformations on the quarks, one will obtain a CKM matrix with the same number of parameters as in the SM. Equation (5.7) states that quark masses are suppressed *at least* by the spurion factor $\xi\eta$, but in fact, they can be smaller. Higher-order mass terms compatible with the symmetry S_{nat} are also possible, starting at the order $\mathcal{O}\left(p^1\xi^1\eta^3\right)$. Accordingly, equation (5.7) should actually be understood as

$$M_{u,d} = \xi\eta\left(\mu_0^{u,d} + \mu_1^{u,d}\eta^2 + \ldots\right), \quad (5.9)$$

where $\mu_0^{u,d}$ and $\mu_1^{u,d}$ are three-by-three matrices reflecting the pattern of flavor symmetry breaking in quark masses. In particular, the top mass should arise from the leading term in (5.9), i.e. it should be expected of the order $\mathcal{O}(\xi\eta)$

$$m_t = \xi\eta\Lambda_Q, \quad (5.10)$$

where Λ_Q is an a priori unspecified large scale related to the characteristic scale of our LEET, $\Lambda_{\text{w}} \sim 4\pi f \sim 3\,\text{TeV}$. Lighter quark masses (including m_b) might well be of a higher order, starting by $\xi\eta^3$, and they can also

receive radiative contributions from loops involving the top quark —see also Section VI.

As mentioned in Section III A 3, consistency of the low-energy power counting for a fermion propagator inside loops requires fermion masses to count as $\mathcal{O}\left(p^1\right)$ or smaller. The above discussion of quark masses therefore suggests a relation between spurion and momentum expansion, specified by the counting rule

$$\xi\eta \; = \; \frac{m_t}{\Lambda_Q} \; = \; \mathcal{O}\left(p^1\right). \qquad (5.11)$$

B. Lepton masses and lepton-number violation

1. Dirac mass terms

The Dirac mass terms for leptons can be written in full analogy with the quark mass term (5.1). We give it here to set up the notation

$$\begin{aligned}\mathcal{L}\left(p^1\xi^1\eta^1\right)_{\text{leptons}} \; &= \; -\mu^{\nu}_{ij}\overline{\ell^i_L}\mathcal{X}^{\dagger}\Sigma\mathcal{Y}_u\ell^j_R \\ &\quad -\mu^{e}_{ij}\overline{\ell^i_L}\mathcal{X}^{\dagger}\Sigma\mathcal{Y}_d\ell^j_R + \text{h.c.}\end{aligned} \qquad (5.12)$$

Here, ℓ^i_L and ℓ^i_R denote left and right-handed lepton doublets, respectively. Upon defining the unitary gauge components of a doublet as for quarks in Section V A, one sees that the Dirac mass term (5.12) can be re-expressed as

$$\begin{aligned}\mathcal{L}\left(p^1\xi^1\eta^1\right)_{\text{leptons}} \; &= \; -\overline{\nu_L}M_\nu\nu_R \\ &\quad -\overline{e_L}M_e e_R + \text{h.c},\end{aligned} \qquad (5.13)$$

where the matrices M_ν and M_e are of a form analogous to (5.7), i.e. they are $\mathcal{O}\left(\xi\eta\right)$ or smaller.

2. Majorana mass terms

In this section, and in the following ones, we work using the spurion $\mathcal{Z}$ defined in (4.34), as opposed to $\mathcal{Z}_{\text{II}}$ (4.58) or $\mathcal{Z}_{\text{III}}$ (4.59). The two latter cases will be considered in Section V C 3.

The Dirac mass term (5.12) is not the only Lorentz-invariant bilinear form of leptons (which are characterized by $B-L=-1$). The unsuppressed $\mathcal{O}\left(p^1\right)$ operator (3.29) with $\Delta L = 2$ is forbidden by the symmetry S_{nat}: in particular, it is custodial symmetry breaking. On the other hand, making use of the spurion $\mathcal{Z}$, one can construct $\Delta L = 2$ operators that are S_{nat}-invariant. These will in turn allow us to construct $(B-L)$-violating operators: this is inevitable in the LEET. In the *right-handed sector*, one can write the $\mathcal{O}\left(p^2\eta^2\zeta_{\text{I}}^2\right)$ operator

$$\begin{aligned}\mathcal{L}\left(p^1\eta^2\zeta_{\text{I}}^2\right)_{\text{leptons}} \; &= \; -\mu^{R}_{ij}\overline{\ell^i_R}\mathcal{Y}_u^{\dagger}\mathcal{Z}\mathcal{Y}_d\left(\ell^j_R\right)^c \\ &\quad +\text{h.c.}\end{aligned} \qquad (5.14)$$

Conjugation for fermion doublets was defined in equation (2.10). In the standard gauge, equation (5.14) reduces to the right-handed neutrino Majorana mass

$$\mathcal{L}\left(p^1\eta^2\zeta_{\text{I}}^2\right)_{\text{leptons}} \overset{\text{s.g.}}{=} -\overline{\nu_R}M_R\left(\nu_R\right)^c + \text{h.c}, \qquad (5.15)$$

where

$$M_R \; = \; \zeta_{\text{I}}^2\eta^2\mu^R. \qquad (5.16)$$

In the left-handed sector, the minimal power of spurions needed to construct S_{nat}-invariant Majorana masses involves two spurions $\mathcal{X}$ in addition to $\mathcal{Z}$

$$\begin{aligned}\mathcal{L}\left(p^1\xi^2\zeta_{\text{I}}^2\right)_{\text{leptons}} \; &= \; -\mu^{L}_{ij}\left(\overline{\ell^i_L}\mathcal{X}^{\dagger}\Sigma\mathcal{Z}\Sigma^{\dagger}\mathcal{X}\left(\ell^j_L\right)^c\right) \\ &\quad +\text{h.c.}\end{aligned} \qquad (5.17)$$

Hence, in the standard gauge, the left-handed neutrino Majorana mass term

$$\mathcal{L}\left(p^1\xi^2\zeta_{\text{I}}^2\right)_{\text{leptons}} \; = \; -\overline{\nu_L}M_L\left(\nu_L\right)^c + \text{h.c}, \qquad (5.18)$$

is of the order $\mathcal{O}\left(p^1\xi^2\zeta_{\text{I}}^2\right)$ with

$$M_L \; = \; \zeta_{\text{I}}^2\xi^2\mu^L. \qquad (5.19)$$

At his stage it is worth stressing that the existence of the $\Delta L = 2$ operators (5.14) and (5.17) and the hierarchy of spurion powers in the corresponding Majorana neutrino masses (cf. (5.16) and (5.19)) are a necessary consequence of the symmetry reduction $S_{\text{nat}} \to S_{\text{red}} = \text{SU}\left(2\right)_L \times \text{U}\left(1\right)_Y$ by means of spurions. In particular, the spurion $\mathcal{Z}$ responsible for the selection of the $\text{U}\left(1\right)_Y$ subgroup (see FIG. 1), controls —via the parameter ζ_{I} — the strength of all $\Delta L = 2$ operators. This leads us to the expectation that ζ_{I} may be much smaller than ξ and η, which are themselves smaller than 1.

C. Smallness of neutrino masses

The Majorana masses for the neutrinos are naturally smaller than the (Dirac) masses of charged fermions, since they involve powers of ζ_{I} in addition to being quadratic in ξ, η. This is not true of the neutrino Dirac masses stemming from (5.12), which would *a priori* be of the order of those of the quarks or charged leptons. On the other hand, we cannot invoke here any see-saw mechanism, since the factor ζ_{I}^2 appearing in the right-handed neutrino mass term (5.16) cannot be assumed large, without ruining our LEET approach.

Note that, if the LEET has to involve an approximate custodial symmetry originating from the right-handed isospin, it should contain a doublet partner ν_R of e_R with a mass $m_{\nu_R} \ll \Lambda_{\text{w}} \sim 3\,\text{TeV}$. Hence, we are facing the problem of a natural suppression of the *neutrino Dirac*

mass matrix M_D in equation (5.13) [11]. It turns out that the spurion framework offers a simple solution to this problem, which we are now going to describe.

1. Specificity of ν_R

The particularity of the three right-handed lepton doublets ℓ^i_R is that they transform under $S_{\rm nat}$ exactly as the spurion doublet ϕ_G defined in (4.62), implying that

$$N^i_R \;\equiv\; \phi^\dagger_G \ell^i_R, \tag{5.20}$$

is invariant under $S_{\rm nat}$. Furthermore, if one sticks to the leading order $\mathcal{O}\left(p^2\right)$ without explicit powers of spurions described by the lagrangian (4.11), the N^i_R satisfy the free massless Dirac equation

$$\begin{aligned}\gamma^\mu \partial_\mu N^i_R \;&=\; \gamma^\mu \left\{ \left(D_\mu \phi^\dagger_G\right) \ell^i_R + \phi^\dagger_G D_\mu \ell^i_R \right\} \\ &=\; 0,\end{aligned} \tag{5.21}$$

as long as the constraint $D_\mu \phi_G = 0$ stemming from (4.21) and the definition (4.62) holds. This in turn implies that the $\mathrm{U}(3)^\nu_R$ subgroup of the classical flavor horizontal symmetry $S^{\rm extended}_{\rm flavor}$ (4.72) generated by

$$\eta^2 Q^{ij}_{R\nu} \;\equiv\; \int \mathrm{d}^3\boldsymbol{x}\, \overline{\ell^i_R} \gamma^0 \mathcal{Y}^\dagger_u \mathcal{Y}_u \ell^j_R, \tag{5.22}$$

is free of anomalies for all gauge field configurations for which the constraints (4.41-4.43) hold. Indeed, the charges $Q^{ij}_{R\nu}$ (5.22) can be equivalently written in terms of the free (massless) right-handed Weyl spinors N^i_R (5.20)

$$Q^{ij}_{R\nu} \;=\; \int \mathrm{d}^3\boldsymbol{x}\, \overline{N^i_R} \gamma^0 N^j_R. \tag{5.23}$$

Notice that this reasoning does not apply in the left-handed case. Even though one can again define a $S_{\rm nat}$ invariant left-handed spinor

$$N^i_L \;\equiv\; \phi^\dagger_G \mathcal{Y}^\dagger_u \Sigma^\dagger \mathcal{X} \ell^i_L \;\overset{\rm s.g.}{=}\; \xi\eta\nu^i_L, \tag{5.24}$$

it does not satisfy a free Dirac equation, due to the presence of the GB matrix Σ in the definition (5.24).

2. ν_R sign-flip symmetry

At the leading order $\mathcal{O}\left(p^2\right)$ without explicit powers of spurions, we thus have a true symmetry $\mathrm{U}(3)^\nu_R$. We should thus ask how the higher-order terms transform with respect to it. The right-handed Majorana mass $M_R = \zeta^2_{\rm I}\eta^2\mu^R$ (5.16) and the neutrino Dirac mass $M_\nu = \xi\eta\mu^\nu$ both break the $\mathrm{U}(3)^\nu_R$ symmetry. However, we can consider a discrete reflection symmetry

$$\mathbb{Z}_2 \;\subset\; \mathrm{U}(3)^\nu_R, \tag{5.25}$$

acting on doublets ℓ^i_R

$$\ell^i_R \;\longmapsto\; (1 - 2\Pi)\,\ell^i_R, \tag{5.26}$$

where Π appearing in (5.26) is the projector on the neutrino component of the right-handed doublet

$$\eta^2 \Pi \;\equiv\; \mathcal{Y}^\dagger_u \mathcal{Y}_u. \tag{5.27}$$

The transformation (5.26) implies, using the properties of the spurions $\mathcal{Y}_u, \mathcal{Y}_d$ (4.32)

$$\mathcal{Y}_u \ell_R \;\longmapsto\; -\mathcal{Y}_u \ell_R, \tag{5.28}$$
$$\mathcal{Y}_d \ell_R \;\longmapsto\; \mathcal{Y}_d \ell_R. \tag{5.29}$$

In the standard gauge, the reflection (5.26) simply amounts to [12]

$$\nu_R \;\longmapsto\; \nu'_R \;=\; -\nu_R. \tag{5.30}$$

It allows for a non-zero right-handed Majorana mass, but forbids the Dirac mass matrix M_ν in (5.13).

The only mass terms for neutrinos are then those of the Majorana type, as given in (5.15) and (5.18). In our case, the smallness of ν_L masses compared to those of charged fermions is directly related to

$$\zeta^2_{\rm I}\xi^2 \;\ll\; \xi\eta. \tag{5.31}$$

This suppression is efficient if $\zeta_{\rm I}$, which appeared first in these lepton-number violating operators, is small with respect to ξ and η.

As for the right-handed neutrinos, their only mass term (5.15) is proportional to $\zeta^2_{\rm I}\eta^2$. We will try to get information on allowed ν_R masses in Section VIII, using cosmological observations, after we discuss their couplings in Section VI B.

3. Strength of $(B-L)$-violating spurions and neutrino masses

We briefly come back to the two other alternatives mentioned in Section IV B 4. In all three cases the ν_R sign-flip symmetry of Section V C 2 has to be used, and we are left with Majorana masses involving a $\zeta^2_{\rm I}, \zeta^2_{\rm II}$ or

[11] Reference [42] deals with a related situation in which the ν_R are light. Though also justified by naturalness, the solution proposed is different.

[12] Note that the ν_R sign-flip symmetry (5.30) can also be imposed in the case of the SM augmented by ν_R degrees of freedom; it is not specific to the Higgs-less case.

$\zeta^2_{\rm III}$ factor that is absent in the charged fermion Dirac masses. Hence, in all cases, the parameter ζ^2 is at the origin of suppression of neutrino masses relative to charged fermion masses. One can check that this would not be possible if we had taken two spurions carrying the $B-L$ charge as independent.

The difference between the three remaining alternatives lies in the accompanying factors of ξ and η, as can be seen by writing down the right-handed and left-handed neutrino Majorana terms.

- Case I was defined in (4.34) without the explicit label I. The respective mass terms for right and left-handed neutrinos are written in (5.14) and (5.17).
- In case II defined in (4.58), we have the following neutrino Majorana mass terms, using $\mathcal{Y}_d^\dagger \mathcal{Z}_{\rm II} = \mathcal{Z}_{\rm II}\mathcal{Y}_u^\dagger = 0$

$$\mathcal{L}\left(p^1\eta^1\zeta^2_{\rm II}\right)_{\rm leptons} = -\overline{\ell^i_R}\mathcal{Y}_u^\dagger \mathcal{Z}_{\rm II}\left(\ell^j_R\right)^c + {\rm h.c}, \quad (5.32)$$

$$\mathcal{L}\left(p^1\xi^2\eta^1\zeta^2_{\rm II}\right)_{\rm leptons} = -\overline{\ell^i_L}\mathcal{X}^\dagger\Sigma\mathcal{Z}_{\rm II}\mathcal{Y}_d^\dagger\Sigma^\dagger\mathcal{X}\left(\ell^j_L\right)^c + {\rm h.c.} \quad (5.33)$$

- In case III defined by (4.59), and using $\mathcal{Y}_d\mathcal{Z}_{\rm III} = \mathcal{Z}_{\rm III}\mathcal{Y}_u^\dagger = 0$

$$\mathcal{L}\left(p^1\zeta^2_{\rm III}\right)_{\rm leptons} = -\overline{\ell^i_R}\mathcal{Z}_{\rm III}\left(\ell^j_R\right)^c + {\rm h.c}, \quad (5.34)$$

$$\mathcal{L}\left(p^1\xi^2\eta^2\zeta^2_{\rm III}\right)_{\rm leptons} = -\overline{\ell^i_L}\mathcal{X}^\dagger\Sigma\mathcal{Y}_u\mathcal{Z}_{\rm III}\mathcal{Y}_d^\dagger\Sigma^\dagger\mathcal{X}\left(\ell^j_L\right)^c + {\rm h.c.} \quad (5.35)$$

We see that the consequence of the different choices of Section IV B 4 on the ratio of left-handed to right-handed neutrino masses are physical, since we have in the various cases the following estimates

$$m^{\rm I}_{\nu_R} \sim \left(\frac{\eta}{\xi}\right)^2 m_{\nu_L}, \quad (5.36)$$

$$m^{\rm II}_{\nu_R} \sim \left(\frac{1}{\xi}\right)^2 m_{\nu_L}, \quad (5.37)$$

$$m^{\rm III}_{\nu_R} \sim \left(\frac{1}{\xi\eta}\right)^2 m_{\nu_L}. \quad (5.38)$$

Note that the heaviest ν_L must be heavier than $\sqrt{\Delta m^2_{\rm atm}} \sim 0.05\,{\rm eV}$, but is also constrained to lie below one eV by cosmological observations (see Sections VIII and particularly VIII B 4), i.e. for our purposes, we can consider it to be experimentally known —up to one order of magnitude. The unknown in relations (5.36-5.38) is then m_{ν_R}: it depends on which of the scenarios I, II or III is realized, whereas m_{ν_L} is assumed to be the physical one. This is the reason why we have written m_{ν_L} without an index I, II, III in the right hand-side of (5.36-5.38). In Section VIII, we will turn to cosmological observations in order to constrain m_{ν_R}.

VI. NON-STANDARD COUPLINGS INDUCED BY SPURIONS

In this section, we describe what could be the next-to-leading order (NLO) of our low-energy expansion. According to the power-counting formula (3.28), loops calculated with lowest order $\mathcal{O}\left(p^2\right)$ vertices, as described by the lagrangian (4.11), start to contribute at chiral order $\mathcal{O}\left(p^4\right)$. Here, we classify and explicitly construct all vertices of chiral order $\mathcal{O}\left(p^2\right)$ or $\mathcal{O}\left(p^3\right)$ which involve (a minimal number of) spurions. Such non-standard vertices will be suppressed by a power of spurions with respect to tree SM contributions. On the other hand, they could be more important than loop effects of the leading order (LO) $\mathcal{O}\left(p^2\right)$ lagrangian, which appear at order $\mathcal{O}\left(p^4\right)$ and hence are —at least formally— more suppressed by the chiral counting rules. A quantitative study of available data, motivated by this classification of NLO contributions, will be presented separately [22].

To organize the discussion, we gather operators of a given order $\mathcal{O}\left(p^\alpha\xi^\beta\eta^\gamma\right)$ according to the value of

$$\kappa = \alpha + \frac{\beta+\gamma}{2}, \quad (6.1)$$

reflecting the possible relation (5.11) between momentum and spurion power-counting. Equation (6.1) presumes that the spurions ξ and η are of a comparable size. As for the spurion $\zeta_{\rm I}$, its presence signals lepton-number violation $\Delta L = 2$. With this in mind, the same counting rule (6.1) can be separately used both in lepton number conserving and LNV sectors.

A. Lepton-number conserving fermion couplings

The first non-standard vertices between fermions and vector bosons appear in the LEET at the level $\kappa = 3$. They are $\mathcal{O}\left(p^2\xi^2\right)$ and $\mathcal{O}\left(p^2\eta^2\right)$. This suggests that, in the Higgs-less LEET, certain vertex corrections could be more important than oblique ones. The latter are discussed in Section VI C, where it will be shown that oblique corrections require $\kappa \geqslant 4$, and receive contributions from loops of the $\mathcal{O}\left(p^2\right)$ lagrangian (4.11).

Let us first concentrate on vector and axial vertices for quarks. We adopt the following generic parametrization of effective left-handed and right-handed couplings

$$\mathcal{L}_{\rm quarks} = eJ^\mu A_\mu + \frac{e}{2cs}\left\{\overline{u_L}\mathcal{F}_L\left(u,u\right)\gamma^\mu u_L + \overline{d_L}\mathcal{F}_L\left(d,d\right)\gamma^\mu d_L\right\}Z_\mu + \frac{e}{2cs}\left\{\overline{u_R}\mathcal{F}_R\left(u,u\right)\gamma^\mu u_R + \overline{d_R}\mathcal{F}_R\left(d,d\right)\gamma^\mu d_R\right\}Z_\mu$$
$$+\frac{e}{\sqrt{2}s}\left(\left\{\overline{u_L}\mathcal{F}_L\left(u,d\right)\gamma^\mu d_L + \overline{u_R}\mathcal{F}_R\left(u,d\right)\gamma^\mu d_R\right\}W^+_\mu + \text{h.c}\right). \tag{6.2}$$

In the above, J^μ is unchanged with respect to the SM, since $\mathrm{U}\left(1\right)_Q$ is unbroken. The couplings $\mathcal{F}_{L,R}$ are three-by-three matrices in generation space. They describe the breaking pattern of the flavor symmetry (4.72), and departure from universality. For leptons, we use the same conventions as in (6.2) after a systematic substitution

$$u \longmapsto \nu, \quad d \longmapsto e. \tag{6.3}$$

1. Flavor structure at the NLO

As already pointed out at the beginning of Section V, successive spurion orders may introduce a hierarchical structure of $S_{\rm flavor}$ symmetry breaking. Within the SM, all direct and indirect effects of flavor symmetry breaking originate in the fermion mass matrix, and they are transmitted via loops involving internal fermion lines: at tree level, the six effective coupling matrices $\mathcal{F}$ are proportional to the unit matrix in an appropriate flavor basis.

Loop-induced flavor symmetry breaking effects can be parametrized by effective $\mathcal{F}$ matrices, which are at least quadratic in fermion masses, since two mass insertions are needed to preserve chirality. According to Section V A, this means that the flavor asymmetry and the non-universality in the couplings (6.2) induced by loops would contribute by terms of order at least $\mathcal{O}\left(\xi^2\eta^2\right)$, i.e. by operators of order $\mathcal{O}\left(p^2\xi^2\eta^2\right)$ or $\kappa = 4$.

In the LEET, a genuine flavor symmetry breaking could already appear at the tree $\kappa = 3$ level, and FCNCs could then be generated already at the NLO. In order to eliminate the latter, it would be sufficient that the mass matrices M_u, M_d, M_e and the neutral current effective couplings $\mathcal{F}_{L,R}\left(f,f\right)$ satisfy the conditions $\left(f = u,d,e\right)$

$$\left[\mathcal{F}_L\left(f,f\right), M_f M_f^\dagger\right] = 0, \tag{6.4}$$
$$\left[\mathcal{F}_R\left(f,f\right), M_f^\dagger M_f\right] = 0. \tag{6.5}$$

M_f and $\mathcal{F}_{L,R}\left(f,f\right)$ would then be diagonalized in the same flavor basis, as it happens e.g. within the minimal flavor violation scheme [43]. However, it is not straightforward to interpret equations (6.4-6.5) as a consequence of a symmetry. For this reason, we adopt a stronger but simpler assumption than (6.4-6.5) or minimal flavor violation: we assume that, including the LO and NLO orders, all flavor symmetry breaking can be transformed from vertices to the fermion mass matrix. In particular, there exists a "flavor-symmetric basis" in which the flavor symmetry (4.72) generated by the (conserved) charges (4.67-4.69) is manifest: in the flavor-symmetric basis, all six matrices $\mathcal{F}$ of equation (6.2) are multiple of the unit matrix. We refer to this hypothesis as "soft flavor violation" (SFV), by analogy with the standard terminology used in the renormalizable case. At the NLO, SFV excludes both the FCNCs and a violation of universality. In the basis in which mass matrices are diagonal, SFV amounts to $\left(f = u,d\right)$

$$\mathcal{F}^{ij}_{L,R}(f,f) = \delta^{ij}F_{L,R}\left(f,f\right) + \text{NNLO}, \tag{6.6}$$

and, for the charged currents

$$\mathcal{F}^{ij}_{L,R}(u,d) = \left(V^{L,R}_{\rm CKM}\right)^{ij}F_{L,R}(u,d) + \text{NNLO}. \tag{6.7}$$

Here, $V^L_{\rm CKM}$ is the CKM matrix and $V^R_{\rm CKM}$ its right-handed analogue: denoting by U_L, U_R, D_L and D_R the $\mathrm{U}\left(3\right)$ transformations of u^i_L, u^i_R, d^i_L and d^i_R respectively, to the basis in which M_u and M_d are diagonal, one has

$$V^L_{\rm CKM} = U_L D_L^\dagger, \tag{6.8}$$
$$V^R_{\rm CKM} = U_R D_R^\dagger. \tag{6.9}$$

By construction, both $V^L_{\rm CKM}$ and $V^R_{\rm CKM}$ are unitary.

Consequences of the hypothesis of SFV in the lepton sector are similar to the case of quarks. They will be discussed shortly.

2. The left-handed quark sector

We now return to the formalism of LEET in order to show how NLO vertices of the type (6.2) can be explicitly constructed with spurions when we require the full $S_{\rm nat}$ symmetry. Since the result is different in the left and right-handed sectors, the two cases will be presented separately.

Let χ_L denote a generic left-handed fermion doublet ($\chi = q$ or ℓ). For each pair of family indices (i,j), there is a single lepton-number conserving invariant bilinear in χ_L that contains one covariant derivative and is independent of the $\mathcal{O}\left(p^2\right)$ SM coupling $\mathrm{i}\overline{\chi_L}\gamma^\mu D_\mu\chi_L$

$$L^{ij}_{\rm NS} = \mathrm{i}\overline{\chi^i_L}\gamma^\mu\mathcal{X}^\dagger\left(\Sigma D_\mu\Sigma^\dagger\right)\mathcal{X}\chi^j_L. \tag{6.10}$$

Indeed, (6.10) represents the minimal spurion insertion into equation (3.32) that is necessary to restore the $S_{\rm nat}$ symmetry. The operator (6.10) is $\mathcal{O}\left(p^2\xi^2\right)$ and therefore has $\kappa = 3$ as defined in (6.1). Consequently, (6.10)

contributes before loops and represents the unique non-standard NLO vertex of left-handed quarks. Using the constraints (4.41-4.43) and the variables introduced in (5.2-5.6) in the standard gauge, (6.10) can be written as

$$L_{\rm NS}^{ij} \overset{\rm s.g.}{=} -\xi^2 \frac{e}{2cs}\left\{\overline{u_L^i}\gamma^\mu Z_\mu u_L^j - \overline{d_L^i}\gamma^\mu Z_\mu d_L^j + \sqrt{2}c\left(\overline{u_L^i}\gamma^\mu W_\mu^+ d_L^j + \text{h.c.}\right)\right\}. \quad (6.11)$$

One observes that the NLO modifications of the left-handed vertex still remain predictive: it is expressed by a single three-by-three matrix in generation space. Comparing with the general phenomenological representation (6.2), it is seen then the NLO contributions $\delta\mathcal{F}$ to the phenomenological matrices $\mathcal{F}$ are related as follows

$$\delta\mathcal{F}_L(u,u) = -\delta\mathcal{F}_L(d,d) = \delta\mathcal{F}_L(u,d). \quad (6.12)$$

Incorporating the hypothesis of SFV at NLO (6.6-6.7), one can summarize left-handed quark couplings including LO and NLO as

$$F_L(u,u) = \left(1-\frac{4}{3}s^2-\xi^2\lambda_L\right)+\text{NNLO}, \quad (6.13)$$

$$F_L(d,d) = \left(-1+\frac{2}{3}s^2+\xi^2\lambda_L\right)+\text{NNLO}, \quad (6.14)$$

$$F_L(u,d) = \left(1-\xi^2\lambda_L\right)+\text{NNLO}. \quad (6.15)$$

At NLO, the left-handed couplings of quarks are described by a single additional constant $\xi^2\lambda_L$, compared to the tree-level SM. There is no modification of the vector electromagnetic coupling at NLO.

3. *The right-handed quark sector*

In order to make an invariant with respect to $S_{\rm nat}$ out of right-handed doublets χ_R^i, instead of (6.10), one needs the spurion $\mathcal{Y}_u$, or its conjugate $\mathcal{Y}_d$. As a result, a single NLO $\mathcal{O}\left(p^2\eta^2\right)$ —i.e. $\kappa=3$ — left-handed invariant (6.10) now becomes three independent invariants $(a,b=u,d)$

$$R_{\rm NS}^{ij}(a,b) = {\rm i}\overline{\chi_R^i}\gamma^\mu\mathcal{Y}_a^\dagger\Sigma^\dagger\left(D_\mu\Sigma\right)\mathcal{Y}_b\chi_R^j. \quad (6.16)$$

Hence, NLO modifications of SM couplings of right-handed quarks is characterized by three parameters $\eta^2\lambda_R^u, \eta^2\lambda_R^d$ and $\eta^2\kappa_R$. Explicitly, using the notation of Section VI A 1, one has in the mass-diagonal flavor basis

$$F_R(u,u) = -\frac{4}{3}s^2+\eta^2\lambda_R^u, \quad (6.17)$$

$$F_R(d,d) = \frac{2}{3}s^2-\eta^2\lambda_R^d, \quad (6.18)$$

$$F_R(u,d) = \eta^2\kappa_R, \quad (6.19)$$

collecting the LO (SM) and NLO contributions. The last equation (6.19) represents the charged right-handed quark currents, which is predicted to appear at the NLO order $\mathcal{O}\left(p^2\eta^2\right)$. Notice that (6.19) arises multiplied by the right-handed CKM matrix $V_{\rm CKM}^R$ —c.f. equations (6.2) and (6.7)— which remains completely unknown. Informations on $\eta^2\kappa_R V_{\rm CKM}^R$ can be expected to arise from $\Delta S=2$ and $\Delta S=1$ FCNC processes induced by loops. An important issue to be clarified is the influence of $\eta^2\kappa_R V_{\rm CKM}^R$ on the standard determination of the CKM matrix elements $\left(V_{\rm CKM}^L\right)^{ij}$.

B. Lepton sector and the interactions of quasi-sterile ν_R

It is straightforward to extend the previous analysis to leptons and extract the lepton-number conserving couplings up to and including the NLO. Here we assume that the hypothesis of SFV applies both for quarks and for leptons. For left-handed couplings, one gets from equation (6.10), in the basis in which both electron and ν_L masses are diagonal

$$\mathcal{F}_L^{ij}(\nu,\nu) = \delta^{ij}\left(1-\xi^2\rho_L\right)+\text{NNLO}, \quad (6.20)$$

$$\mathcal{F}_L^{ij}(e,e) = \delta^{ij}\left(-1+2s^2+\xi^2\rho_L\right)+\text{NNLO}, \quad (6.21)$$

$$\mathcal{F}_L^{ij}(\nu,e) = \left(V_{\rm MNS}^L\right)^{ij}\left(1-\xi^2\rho_L\right)+\text{NNLO}, \quad (6.22)$$

where $V_{\rm MNS}^L$ is the MNS matrix describing the transformation between flavor and mass basis for left-handed neutrinos and electrons.

An important distinction between quark and lepton couplings arises in the right-handed sector: whereas they were allowed for quarks (6.19), charged right-handed currents are forbidden for leptons, due to the ν_R sign-flip symmetry (5.26) introduced in the previous Section. Neither are such couplings generated by loops, as long as the discrete symmetry is assumed to be exact [13]. Since it also forbids Dirac mass term for neutrinos, the consequence of the ν_R sign-flip symmetry is that the right-handed neutrinos do not interact (or mix) with their left-handed counterparts. Therefore, there can be no oscillations between the two, and the so-called LSND anomaly [44] cannot be explained in this way. The right-handed lepton couplings read off from equation (6.16) can be written as

$$\mathcal{F}_R^{ij}(\nu,\nu) = \delta^{ij}\eta^2\rho_R^\nu+\text{NNLO}, \quad (6.23)$$

$$\mathcal{F}_R^{ij}(e,e) = \delta^{ij}\left(2s^2-\eta^2\rho_R^e\right)+\text{NNLO}, \quad (6.24)$$

$$\mathcal{F}_R^{ij}(\nu,e) = 0. \quad (6.25)$$

Note that the three right-handed neutrinos are expected to be lighter than the scale $\Lambda_{\rm w}\sim 3\,\text{TeV}$, oth-

[13] Though natural and self-consistent, the assumption that this symmetry is exact is merely dictated by simplicity, and could therefore be relaxed. It could be easily falsified by a detection of charged leptonic right-handed currents.

erwise they would not belong to the LEET. This is emphasized in (5.16) by the explicit power of ζ_{I}^2 appearing in their (Majorana) masses. Therefore, we would expect them to be light enough in order to be pair-produced in Z^0 decays. Since the ν_R carry no $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ charge, however, their couplings to electroweak vector bosons only come with explicit powers of spurions: the right-handed neutrinos decouple in the limit where the spurions are sent to zero. Their couplings are therefore suppressed with respect to weak interactions: they are "quasi-sterile", but do have interactions with Z^0 due to the $\mathcal{O}\left(p^2\eta^2\right)$ operator yielding (6.23), which we rewrite in full here

$$\mathcal{L}\left(p^2\eta^2\right) \;=\; \rho_R^{\nu}\eta^2 \frac{e}{2cs}\overline{\nu_R}\gamma^{\mu}\nu_R Z_{\mu}. \qquad (6.26)$$

If the suppression factor η is of order 0.1 or smaller, we find that the contribution to the Z^0 invisible width is smaller than the errors on this observable [45]. We will assume this to be true throughout the sequel. If this were not the case, then we would conclude that the Higgs-less scenarios described by the present effective theory including light quasi-sterile right-handed neutrinos are excluded. One would then have to find an alternative way of ensuring an approximate custodial symmetry without introducing right-handed neutrinos at all. As far as we know, this is not excluded: we have only considered the simplest possibility of interpreting the right isospin as the origin of custodial symmetry [19]. Since such quasi-sterile ν_R are not excluded by particle physics experiments, we will turn to cosmological observations in Section VIII.

C. Comment on oblique corrections

We have considered above the LO and NLO of the Higgs-less LEET which should both be treated at tree level: they involve operators with respectively $\kappa = 2$ and $\kappa = 3$. The set of operators usually collected under the name of "oblique corrections" has $\kappa = 4$ in this framework and is therefore *not* the main "deviation from the SM": these operators appear only at NNLO, i.e. at the same level as loops. This is the reason why they should be considered after the non-universal couplings of Sections VI A and VI B. In this case, we have to give up the formalism of oblique corrections, which was specifically designed to take into account cases where the main effect from new physics would occur in two-point function of gauge bosons [14]. The outcome is that one must resort to observables, as advocated for instance in [49]: in brief, oblique corrections are not the first corrections to be looked for in Higgs-less scenarios, and they cannot be discussed independently of loops.

Having stressed that, we mention the following $\kappa = 4$ operators which all reduce, in the standard unitary gauge, to the same operator usually associated with the oblique S parameter

$$\left\langle G_{L\mu\nu}\mathcal{X}^{\dagger}\Sigma\left(\mathcal{Y}_u+\mathcal{Y}_d\right)G_R^{\mu\nu}\left(\mathcal{Y}_u^{\dagger}+\mathcal{Y}_d^{\dagger}\right)\Sigma^{\dagger}\mathcal{X}\right\rangle \overset{\text{s.g.}}{=} \xi^2\eta^2 b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right), \qquad (6.27)$$

$$\left\langle G_{L\mu\nu}\mathcal{X}^{\dagger}\Sigma\left(\mathcal{Y}_u-\mathcal{Y}_d\right)G_R^{\mu\nu}\left(\mathcal{Y}_u^{\dagger}-\mathcal{Y}_d^{\dagger}\right)\Sigma^{\dagger}\mathcal{X}\right\rangle \overset{\text{s.g.}}{=} \xi^2\eta^2 b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right), \qquad (6.28)$$

$$\left\langle \Gamma_{L\mu\nu}\Sigma\left(\mathcal{Y}_u+\mathcal{Y}_d\right)G_R^{\mu\nu}\left(\mathcal{Y}_u^{\dagger}+\mathcal{Y}_d^{\dagger}\right)\Sigma^{\dagger}\right\rangle \overset{\text{s.g.}}{=} g\eta^2 b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right), \qquad (6.29)$$

$$\left\langle \Gamma_{L\mu\nu}\Sigma\left(\mathcal{Y}_u-\mathcal{Y}_d\right)G_R^{\mu\nu}\left(\mathcal{Y}_u^{\dagger}-\mathcal{Y}_d^{\dagger}\right)\Sigma^{\dagger}\right\rangle \overset{\text{s.g.}}{=} g\eta^2 b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right), \qquad (6.30)$$

$$\left\langle G_{L\mu\nu}\mathcal{X}^{\dagger}\Sigma\Gamma_R^{\mu\nu}\Sigma^{\dagger}\mathcal{X}\right\rangle \overset{\text{s.g.}}{=} g'\xi^2 b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right), \qquad (6.31)$$

$$\left\langle \Gamma_{L\mu\nu}\Sigma\Gamma_R^{\mu\nu}\Sigma^{\dagger}\right\rangle \overset{\text{s.g.}}{=} gg' b_{\mu\nu}\left(sA^{\mu\nu}+cZ^{\mu\nu}\right). \qquad (6.32)$$

These operators should be compared with loops generated by the lagrangian (4.11), which is also different from the SM, since it does not contain a Higgs particle. Also, universal contributions from the operators mentioned in Sections VI A and VI B will modify the predictions for the observables from which the S parameter should be extracted.

Another oblique parameter is T: it represents a direct tree contribution to the Z^0 mass not affecting the $W^{\pm}$ mass. In our LEET, the size of such custodial-breaking effects are parameterized by the spurion $\mathcal{Y}$ (i.e. the parameter η), via the $\mathcal{O}\left(p^2\eta^4\right)$ operator

$$\Lambda^2\left\langle \Sigma^{\dagger}D_{\mu}\Sigma\left(\mathcal{Y}_u\mathcal{Y}_u^{\dagger}-\mathcal{Y}_d\mathcal{Y}_d^{\dagger}\right)\right\rangle^2 \overset{\text{s.g.}}{=} -\Lambda^2\eta^4\frac{e^2}{c^2s^2} \times Z_{\mu}Z^{\mu}. \qquad (6.33)$$

[14] We note that, for the special case of 5D Higgs-less models, it has been stressed in [46] that non-oblique corrections were relevant, even though they were of the type called "universal" in [47]. Such corrections may in fact even be required in 5D Higgs-less models, to compensate in some observables for the non-zero value of the S parameter [21, 48].

D. Prefactors and low-energy constants

So far, we have merely concentrated on the ordering of operators in the effective lagrangian according to powers of momenta and spurions. This is needed for the formal definition of a consistent quantum theory in a systematic low-energy expansion. When expressed in the standard gauge, each operator already contains a definite power of g, g' and of spurion factors ξ, η —c.f. the NLO operators (6.10) and (6.16), or the NNLO oblique operators (6.27-6.32). Actually, each of these operators will enter the lagrangian multiplied by a LEC, which is not determined by symmetry arguments. Here, we would like to comment about the possible order of magnitude estimate of such LECs, keeping in mind that, though constrained by consistency of the order-by-order renormalization, such estimates can only be validated by a detailed quantitative analysis.

There are in fact three distinct classes of operators in the effective theory: those that involve only elementary fields, those that involve only composite fields, and finally those that mix elementary and composite fields. In the first class, prefactors can be estimated since the elementary sector is weakly-interacting.

For the second class, the precedent of χPT is of great help to estimate factors of $1/\left(16\pi^2\right)$ or f^2/Λ^2. In particular, the constant in front of the operator (6.32) corresponds to L_{10} in χPT [6]. In both effective theories (χPT and the present one), the formal counting puts this operator at the same level as one-loop graphs. This is indeed realized numerically if the LEC in front of (6.32) is of order the loop factor $1/16\pi^2$, as observed experimentally for the case of χPT (for a reasonable renormalization scale). Such a QCD-like contribution to the S parameter has the right magnitude, but the wrong sign, to compensate for the absence of diagrams with a Higgs [15]. Still, this is not the end of the story here, since operators of the third class also contribute to the S parameter.

As for the operators belonging to the third class, which mix the elementary and composite sector and therefore involve spurions, their prefactors can be estimated only by assuming consistency with the power counting of the effective theory. Consider for instance the operators (6.27, 6.28), which have not been taken into account in the literature: they have no equivalent in χPT (they contain the spurions ξ and η). One can fix a normalization of the spurions by assuming that the two operators (6.27, 6.28) enter the lagrangian with order one prefactors. Comparing the contributions to S of the operators (6.27, 6.28) on one side, and of (6.32) on the other side, and assuming that they are not only of the same formal order, but of the same numerical order, we obtain

$$\xi^2 \sim \frac{g}{4\pi}, \quad \eta^2 \sim \frac{g'}{4\pi}. \tag{6.34}$$

Similarly, one expects that the operators (6.29-6.31) appear with a prefactor of order $1/(4\pi)$. Note that, in this case, the total tree contribution to S is still expected to be of the right order of magnitude to compete with loop contributions.

Since we have arbitrarily normalized the spurions by assuming order one prefactors in (6.27, 6.28), slightly smaller/larger prefactors might appear in front of other operators, for instance the fermion mass term (5.10) and the non-standard fermion-gauge interactions (6.10, 6.16). In the case of the top mass, this can be equivalently phrased as Λ_Q being different from $\Lambda_{\mathrm{w}} \sim 3.1$ TeV. However, we estimate $\Lambda_Q \sim 4.7$ TeV, not a large relative factor. In the non-standard interactions (6.10, 6.16), we see that a prefactor of order 0.1 would be required to satisfy electroweak tests. Inside the effective theory, we do not know what the origin of this additional suppression could be: the possibility cannot be ruled out by logic alone (one cannot expect the effective theory itself to exclude Higgs-less scenario), and must be investigated in a detailed comparison with experiment .

VII. LEPTON-NUMBER VIOLATION AT THE NLO

The left-handed and right-handed Majorana neutrino masses are the only manifestation of LNV and of the spurion ζ_{I} considered so far. Here we concentrate on other LNV effects. The LEET approach, which presumes a separation of low and high mass scales should allow to consider such effects in full generality without commitment to a particular scenario for physics at the high scale. For definiteness, we stick to the process

$$W^- + W^- \longrightarrow e^- + e^-, \tag{7.1}$$

which can be either real (possibly observable at a high luminosity collider), or virtual. In the latter case, the process (7.1) appears as a sub-diagram of various neutrinoless double beta decays ($0\nu 2\beta$) such as $(A, Z) \longrightarrow (A, Z+2) + e^- + e^-$, rare K decays $K^- \longrightarrow \pi^+ + e^- + e^-$, or their lepton flavor violating variants. There are two distinct problems: i) one must estimate the hadronic matrix elements of the product of the two hadronic currents coupled to the W's of (7.1). This problem is yet another challenge to the non-perturbative QCD phenomenology, and will not be addressed in this work. We are more particularly concerned with the second problem: ii) How could an experimental information about the strength, electron spectrum and polarizations of the elementary process (7.1) be exploited to learn about the mechanism and parameters of LNV. In particular, could it be used to extract an experimental information about neutrino masses and mixings?

A. Tree contributions to $e^- + e^- \longrightarrow W^- + W^-$

Four different types of contributions to the process (7.1) are classified in FIGs. 2-3.

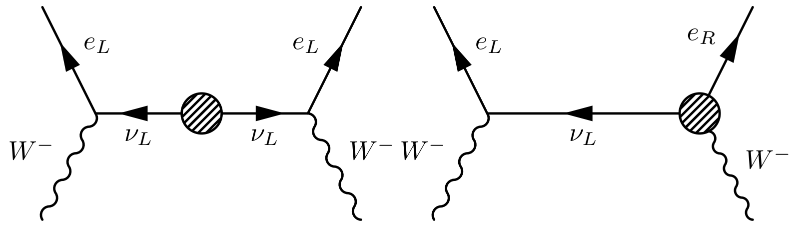

FIG. 2: Majorana mass insertions and odd chirality LNV vertex.

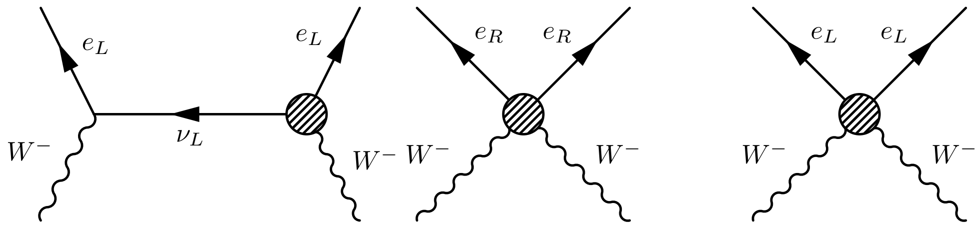

FIG. 3: Magnetic type LNV vertex and LNV contact terms.

Let us first focus on the Majorana mass insertion in the t-channel neutrino exchange shown in FIG. 2. In this contribution, the lepton number violation is *indirect* since it does not involve any one-particle irreducible LNV vertex. It stems from the square of the standard $\Delta L = 0$ interaction term

$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}} \sum_{i,j} V_{\mathrm{MNS}}^{i,j} \left(\overline{\nu_L^i} \gamma^\mu e_L^j \right) W_\mu^+ + \mathrm{h.c}, \quad (7.2)$$

where ν_L^j ($j = 1, 2, 3$) is the j-th left-handed Majorana mass eigenstate and V_{MNS} stands for the MNS mixing matrix. Since the asymptotic field ν_L^j obeys the chirally projected Majorana-Dirac equation [15]

$$\mathrm{i}\gamma^\mu \partial_\mu \nu_L^j = m_j \left(\nu_L^j \right)^c, \quad (7.3)$$

there exists a $\Delta L = 2$ correlation leading to the propagator

$$\left\langle 0 | T \nu_L(x) \overline{(\nu_L)^c}(y) | 0 \right\rangle = m \frac{1-\gamma_5}{2} \times \Delta_F(x-y, m), \quad (7.4)$$

which is represented in FIG. 2 by the line with a shaded circle. The corresponding indirect LNV contribution to $e^i + e^j \longrightarrow W + W$, to $0\nu 2\beta$ decays and to other $\Delta L = 2$ processes is well-known and described in the literature, see for instance [50]. It is entirely given by the Majorana mass matrix element

$$\mu^{i,j} = \sum_k \left(V_{\mathrm{MNS}}^{k,i} \right)^* \left(V_{\mathrm{MNS}}^{k,j} \right)^* m_k. \quad (7.5)$$

[15] In the present case it is not particularly convenient to use Majorana fields $\nu_M = \nu_M^c$ instead of the chiral ones $\gamma_5 \nu_L = -\nu_L$. These are just two equivalent sets of variables related by the transformations $\nu_M = \nu_L + \nu_L^c$, $\nu_L = \frac{1-\gamma_5}{2}\nu_M$, describing the same lagrangian and the same physics.

In the approximation $p^2 \gg m^2$, the contribution to the amplitude $ee \longrightarrow WW$ becomes

$$M_1 = \frac{g^2}{2} \Delta_F(x-y) \left[\overline{e_L}(x) V_{\mathrm{MNS}}^\dagger m V_{\mathrm{MNS}}^* (e_L(y))^c \right] \times W^{-\mu}(x) W_\mu^-(y). \quad (7.6)$$

The diagrams shown in FIGs. 2 and 3 represent different types of *direct contributions* from irreducible LNV vertices. Within the present LEET framework, the existence and strength of the latter depends on their chiral dimension and on their minimal spurion content as dictated by the symmetry properties under S_{nat}. Before we proceed to this analysis (Sections VII B and VII C), it is useful to clarify the chirality structures of possible $\Delta L = 2$ vertices using nothing but Lorentz invariance.

FIG. 2 involves possible *chirality-odd* $\Delta L = 2$ vertex (indicated by a shaded circle) which, in the standard unitary gauge, would be of the form

$$\begin{aligned} \mathcal{L}_{\Delta L=2}^{\mathrm{odd}} &= g \left(\overline{e_R} \epsilon \gamma^\mu (\nu_L)^c \right) W_\mu^- + \mathrm{h.c}, \\ &= -g \left(\overline{\nu_L} \epsilon^T \gamma^\mu (e_R)^c \right) W_\mu^- + \mathrm{h.c}. \quad (7.7) \end{aligned}$$

Interference with the standard coupling (7.2) contains the unsuppressed chirality-odd part of the neutrino propagator

$$\begin{aligned} S_F(x-y,m) &\equiv \left\langle 0 | T \nu_L(x) \overline{(\nu_L)}(y) | 0 \right\rangle \\ &= \frac{1-\gamma_5}{2} \gamma^\mu \partial_\mu \Delta_F(x-y,m), \quad (7.8) \end{aligned}$$

and the contribution to the amplitude $ee \longrightarrow WW$ shown in FIG. 2 becomes

$$M_2 = g^2 \left[\overline{e_L}(x) \gamma^\mu V_{\mathrm{MNS}}^\dagger S_F(x-y,m) \epsilon^T \gamma^\nu (e_R(y))^c \right] \times W_\mu^-(x) W_\nu^-(y). \quad (7.9)$$

For not too small neutrino momenta $p^2 \gg m^2$, the mass dependence of $S_F(x-y,m)$ can be neglected and the coupling matrix in (7.9) reduces to $V^\dagger \epsilon^T$, i.e. it is a priori *independent of neutrino masses*. Comparing (7.6) with (7.9), we note that the indirect contribution of FIG. 2 and the direct one of FIG. 2 lead to different polarization spectra and angular distribution of the two final leptons in $WW \longrightarrow ee$. In principle, they could be distinguished experimentally.

FIG. 3 describes the contribution to $WW \longrightarrow ee$ arising from direct LNV *chirality-even* vertices (shaded circle). Such vertices are of magnetic type $\overline{e_L}\sigma^{\mu\nu}(\nu_L)^c \partial_\mu W_\nu^-$. Using equations of motion, they can be reduced to

$$\mathcal{L}^{\text{even}}_{\Delta L=2} \;=\; \frac{1}{\Lambda} W_\mu^- \overline{e_L}\epsilon' \partial^\mu (\nu_L)^c + \text{h.c}, \qquad (7.10)$$

with its own coupling matrix ϵ'. The vertex (7.10) contains one more power of momentum as compared to the chirality-odd vertex (7.7). It is therefore dimensionally suppressed by the inverse power of an a priori unknown scale Λ. The corresponding contribution to $WW \longrightarrow ee$ (FIG. 3) reads

$$\begin{aligned} M_3 \;=\; & \frac{g}{\Lambda}\left[\overline{e_L}(x)\gamma^\mu V^\dagger_{\text{MNS}}\epsilon'^T S_F(x-y,m)\partial^\nu (e_L(y))^c\right] \\ & \times W_\mu^-(x) W_\nu^-(y). \end{aligned} \qquad (7.11)$$

It has a similar helicity structure as the indirect contribution of FIG. 2, but it differs by the distribution of final lepton momenta. Before one concludes that this direct LNV term can be neglected on the basis of momentum power counting, one has to check the spurion content of ϵ' compared to that of ϵ, and to that of the Majorana mass m.

The direct LNV contact term of FIG. 3 will be discussed in Section VII C 3.

B. The unique $\Delta L = 2$ chirality-odd vertex

In order to infer the degree of suppression of the coupling ϵ and ϵ', one has to go back to the formalism of Section IV. The $\Delta L = 2$ vertices must be constructed out of lepton doublets ℓ_L and ℓ_R, as well as their conjugates $(\ell_{L,R})^c$, of the gauge connections, the GB matrix Σ and the spurions $\mathcal{X}, \mathcal{Y}_{u,d}$ and $\mathcal{Z}$. The vertices must be invariant under the whole group S_{nat} and moreover, they have to respect the $\mathbb{Z}_2$ ν_R *sign-flip symmetry* (5.26). It turns out that there is a single chirality-odd vertex with these properties carrying the minimal chiral order and a minimal number of spurion insertions. The unique result reads

$$\begin{aligned} \mathcal{L}^{\text{odd}}_{\Delta L=2} \;=\; & \sum_{i,j} c_{ij}\overline{\ell^i_R}\mathcal{Y}_d^\dagger \gamma^\mu \left(\Sigma^\dagger D_\mu \Sigma\right)\mathcal{Z}\Sigma^\dagger \mathcal{X}\left(\ell^j_L\right)^c \\ & +\text{h.c.} \end{aligned} \qquad (7.12)$$

It is remarkable that the chain of factors in (7.12) is uniquely determined by the symmetry properties, including the ν_R sign-flip symmetry which plays a crucial role in the above construction. It provides a non-trivial illustration of the spurion formalism at work.

In the standard unitary gauge (7.12) reduces to (7.7) with the coupling matrix ϵ given by

$$\epsilon_{ij} \;=\; \zeta_{\text{I}}^2 \xi\eta c_{ij}, \qquad (7.13)$$

the factor g being contained in $\Sigma^\dagger D_\mu \Sigma$. Compared to the left-handed Majorana mass matrix (5.17)

$$(M_L)_{ij} \;=\; \Lambda\zeta_{\text{I}}^2 \xi^2 \mu^L_{ij}, \qquad (7.14)$$

one observes that the degree of spurion suppression of the indirect and direct LNV contribution is similar as long as $\xi \sim \eta$. A more detailed comparison for the process $WW \longrightarrow ee$ should be made in terms of the matrix elements M_1 and M_2 of (7.6) and (7.9). One finds

$$\frac{M_2}{M_1} \;\sim\; \frac{\langle p\rangle}{\Lambda}\frac{\eta}{\xi}, \qquad (7.15)$$

where $\langle p\rangle$ is the average momentum transfer by the neutrino —c.f. FIG. 2. Λ is the value which the masses of left-handed neutrinos would take in the absence of the spurion suppression factors.

C. Direct LNV chirality-even vertices

We now consider $\Delta L = 2$ vertices of the form

$$\overline{\ell_L} T_L (\ell_L)^c \;\text{ or }\; \overline{\ell_R} T_R (\ell_R)^c. \qquad (7.16)$$

$T_{L,R}$ are matrices made up from gauge fields, GBs Σ and from spurions. They transform under S_{nat} according to

$$T_{L,R} \;\longmapsto\; \mathrm{e}^{\mathrm{i}\alpha} G_{L,R} T_{L,R} G^\dagger_{L,R}. \qquad (7.17)$$

$T_{L,R}$ should contain an even number of γ-matrices. Furthermore, in order to reconcile LNV with the symmetry S_{nat}, $T_{L,R}$ must contain the spurions $\mathcal{Z}$.

Interactions of this type can either contribute to the $\Delta L = 2$ processes discussed before, such as $0\nu 2\beta$ decays, via diagrams shown on FIGs. 3 and 3 or they can generate new independent $\Delta L = 2$ processes, such as the decays $Z \longrightarrow \nu\nu$ or $Z \longrightarrow \overline{\nu\nu}$, or magnetic-type transitions

$$\nu_L \;\longleftrightarrow\; \overline{\nu_L} + \gamma, \quad \nu_R \;\longleftrightarrow\; \overline{\nu_R} + \gamma, \qquad (7.18)$$

between different Majorana mass eigenstates of the left-handed or right-handed neutrinos. In the following, we give an essentially complete list of such chirality-even $\Delta L = 2$ vertices and clarify their degree of suppression by spurions.

1. Right-handed Majorana magnetic coupling

As a consequence of the presumably exact ν_R sign-flip symmetry (5.26), there are no Pauli-Dirac magnetic couplings of neutrinos. Furthermore, due to the Fermi statistic, the diagonal part of Majorana magnetic couplings vanishes, leaving no place for a neutrino magnetic moment to all orders of our LEET. The Majorana *magnetic transition moments* can be constructed in a full analogy with Majorana mass terms of Section V B 2. As in the latter case, the result, in particular the degree of spurion suppression, could be different in the left-handed and right-handed cases.

The lowest order $S_{\rm nat}$-invariant right-handed magnetic vertex merely involves the insertion of the spurion $\mathcal{Z}$

$$\mathcal{L}^{R}_{\rm mag} \;=\; \frac{1}{\Lambda}\overline{\ell^i_R}\mathcal{Y}_u^\dagger \mathcal{Z}\mathcal{Y}_d \sigma^{\mu\nu}\left(\ell^j_R\right)^c B_{\mu\nu} + {\rm h.c.} \quad (7.19)$$

It is of the order $\mathcal{O}\left(p^2\zeta_{\rm I}^2\eta^2\right)$. $B_{\mu\nu}$ is the $\mathrm{U}(1)_{B-L}$ field strength and Λ is a scale reflecting the dimensional suppression. The vertex (7.19) is antisymmetric in the lepton flavor indices $\{i,j\}$. Furthermore, (7.19) exhibits the ν_R sign-flip symmetry automatically: in the standard unitary gauge, it reduces to a single term quadratic in ν_R

$$\mathcal{L}^{R}_{\rm mag} \overset{\rm s.g.}{=} \frac{\zeta_{\rm I}^2\eta^2}{\Lambda}\overline{\nu^i_R}\sigma^{\mu\nu}\left(\nu^j_R\right)^c\left(cA_{\mu\nu} - sZ_{\mu\nu}\right) + {\rm h.c}, \quad (7.20)$$

where $A_{\mu\nu}$ is the electromagnetic field and similarly, $Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$.

The magnetic vertex (7.20), together with the LN-conserving interaction $Z \longrightarrow \overline{\nu_R}\nu_R$ (6.26) completes the picture of residual interactions of quasi-sterile right-handed neutrinos. They are induced by spurions, $\mathcal{O}\left(p^2\eta^2\right)$ in the case of (6.26) and $\mathcal{O}\left(p^2\zeta_{\rm I}^2\eta^2\right)$ in the LNV case (7.20). Note that, with three families, decays such as (7.18) imply that only the lightest ν_R and ν_L are stable. However, the spurion factors in (7.20) are such that, for masses below the expected maximum of order keV, as allowed for m_{ν_R} in case III of Section V C 3, the lifetime of all neutrino species is longer than the age of the universe [16]. Hence, we shall consider that all neutrinos are stable for the purpose of the discussion of their cosmological impact in Section VIII.

2. Left-handed LNV magnetic couplings

We now turn to the magnetic couplings in the left-handed sector. The minimal set of spurions which is needed to restore the invariance under $S_{\rm nat}$ is the same as in the case of left-handed Majorana masses (5.17). The extension of (7.19) reads

$$\mathcal{L}^{L,1}_{\rm mag} \;=\; \frac{1}{\Lambda}\overline{\ell^i_L}\mathcal{X}^\dagger\Sigma\mathcal{Z}\Sigma^\dagger\mathcal{X}\sigma^{\mu\nu}\left(\ell^j_L\right)^c B_{\mu\nu} + {\rm h.c} \quad (7.21)$$

which is $\mathcal{O}\left(p^2\xi^2\zeta_{\rm I}^2\right)$. In the standard unitary gauge, it reduces to

$$\mathcal{L}^{L,1}_{\rm mag} \overset{\rm s.g.}{=} \frac{\zeta_{\rm I}^2\xi^2}{\Lambda}\overline{\nu^i_L}\sigma^{\mu\nu}\left(\nu^j_L\right)^c\left(cA_{\mu\nu} - sZ_{\mu\nu}\right) + {\rm h.c.} \quad (7.22)$$

We see that the spurion factors suppressing the magnetic transition $\nu_L \longrightarrow \overline{\nu_L} + \gamma$ and $\nu_R \longrightarrow \overline{\nu_R} + \gamma$ are the same as for the respective Majorana mass terms. In the left-handed sector, additional LNV magnetic couplings of the order $\mathcal{O}\left(p^2\xi^2\zeta_{\rm I}^2\right)$ are possible. They are obtained inserting into appropriate places of the spurion chain (7.21) the field strength $G_{L\mu\nu} \longmapsto G_L G_{L\mu\nu} G_L^\dagger$ instead of the invariant $B_{\mu\nu}$

$$\mathcal{L}^{L,2}_{\rm mag} \;=\; \frac{1}{\Lambda}\overline{\ell^i_L}G_{L\mu\nu}\mathcal{X}^\dagger\Sigma\mathcal{Z}\Sigma^\dagger\mathcal{X}\sigma^{\mu\nu}\left(\ell^j_L\right)^c + {\rm h.c.} \quad (7.23)$$

In the standard unitary gauge, this vertex reads

$$\mathcal{L}^{L,2}_{\rm mag} \overset{\rm s.g.}{=} \frac{\zeta_{\rm I}^2\xi^2}{\Lambda}\left\{\overline{\nu^i_L}\sigma^{\mu\nu}\left(\nu^j_L\right)^c\left(sA_{\mu\nu} + cZ_{\mu\nu}\right) + \sqrt{2}c\overline{e^i_L}\sigma^{\mu\nu}\left(\nu^j_L\right)^c W_{\mu\nu} + {\rm h.c}\right\}, \quad (7.24)$$

where $W_{\mu\nu} \equiv \nabla_\mu W_\nu - \nabla_\nu W_\mu + \ldots$ is the charged component of the field strength $G_{L\mu\nu}$ expressed in the unitary gauge. The new element with respect to the right-handed case, is the appearance of the charged current vertex $W^-_\mu \longrightarrow e^-_L + \nu_L$. This was not possible in the right-handed case, due to the ν_R sign-flip symmetry. This represents the *chirality-even direct* contribution to the process $WW \longrightarrow e_L e_L$ via the diagram represented in FIG. 3. The order of magnitude estimate of this contribution suggests

$$M_3 \;\sim\; g\frac{m_{\nu_L}}{\Lambda^2}\overline{e_L}\left(e_L\right)^c W^-_\mu W^{-\mu}. \quad (7.25)$$

It appears that this contribution contains one power of g less than the indirect LNV contribution, equation (7.6). Nevertheless, one should have $M_3 \ll M_1$ due to the relative suppression factor $1/g\left\langle p^2\right\rangle/\Lambda^2$ (where $\left\langle p^2\right\rangle$ is the average momentum transfered by the neutrino).

3. Contact contribution to $WW \longrightarrow ee$

We finally consider chirality-even LNV vertices containing two covariant derivatives acting on GB fields Σ [17]. In the right-handed sector such vertices are of the

[16] We thank Émilie Passemar for an explicit check of this estimate.

[17] Terms with derivatives of fermion fields are related by equations of motion to the magnetic-type vertices considered previously.

type

$$N_{RR}^{(a)} = \frac{1}{\Lambda}\overline{\ell_R}\mathcal{Y}_a^{\dagger}\left(\Sigma^{\dagger}D_{\mu}\Sigma\right)\mathcal{Z}\left(\Sigma^{\dagger}D^{\mu}\Sigma\right)\mathcal{Y}_a^{c}\left(\ell_R\right)^{c} + \text{h.c}, \tag{7.26}$$

where $a = u$ or d. In equation (7.26), the occurrence (twice) of the $\mathcal{Y}_a$ spurion guarantees that the the operator is invariant under the ν_R sign-flip symmetry. Since in the standard unitary gauge we have

$$\mathrm{i}\Sigma^{\dagger}D_{\mu}\Sigma \overset{\text{s.g.}}{=} \frac{e}{2cs}\left\{Z_{\mu}\tau^{3} + \sqrt{2}c\left(W_{\mu}^{+}\tau^{+} + W_{\mu}^{-}\tau^{-}\right)\right\}, \tag{7.27}$$

we obtain

$$N_{RR}^{(u)} \overset{\text{s.g.}}{=} -\frac{\zeta_{\mathrm{I}}^{2}\eta^{2}}{\Lambda}\left(\frac{e}{2cs}\right)^{2}\overline{\nu_R}\left(\nu_R\right)^{c}Z_{\mu}Z^{\mu} + \text{h.c} \tag{7.28}$$

whereas for $a = d$, the vertex (7.26) reduces to

$$N_{RR}^{(d)} \overset{\text{s.g.}}{=} -\frac{\zeta_{\mathrm{I}}^{2}\eta^{2}}{2\Lambda}\left(\frac{e}{s}\right)^{2}\overline{e_R}\left(e_R\right)^{c}W_{\mu}^{-}W^{-\mu} + \text{h.c} \tag{7.29}$$

Equation (7.29) represents a direct LNV contact contribution to $WW \longrightarrow ee$, as represented in FIG. 3. It is of order $\mathcal{O}\left(p^{2}\eta^{2}\zeta_{\mathrm{I}}^{2}\right)$, i.e. carries the index $\kappa = 5$, and should normally be suppressed compared to the indirect LNV diagram of FIG. 2. We indeed find

$$\frac{M_4^{RR}}{M_1} \sim \left(\frac{p}{\Lambda}\frac{\eta}{\xi}\right)^{2}. \tag{7.30}$$

In the left-handed sector, the two-derivative vertex similar to (7.26) depicted in FIG. 3 reads

$$N_{LL}^{(a)} = \frac{1}{\Lambda}\overline{\ell_L}\mathcal{X}^{\dagger}\left(D_{\mu}\Sigma\right)\mathcal{Z}\left(D^{\mu}\Sigma^{\dagger}\right)\mathcal{X}\left(\ell_L\right)^{c} + \text{h.c}, \tag{7.31}$$

and in the standard unitary gauge it becomes

$$N_{LL} \overset{\text{s.g.}}{=} \frac{\zeta_{\mathrm{I}}^{2}\xi^{2}}{\Lambda}\left(\frac{e}{2cs}\right)^{2}\left\{\overline{\nu_L}\left(\nu_L\right)^{c}Z_{\mu}Z^{\mu} + 2\sqrt{2}c\overline{e_L}\left(\nu_L\right)^{c}W_{\mu}^{-}Z^{\mu} + \text{h.c.} + \frac{c^{2}}{2}W_{\mu}^{-}W^{-\mu}\overline{e_L}\left(e_L\right)^{c}\right\}. \tag{7.32}$$

It again contains the contact contribution to $WW \longrightarrow e_Le_L$ of the order $\mathcal{O}\left(p^{2}\xi^{2}\eta^{2}\zeta_{\mathrm{I}}^{2}\right)$, i.e. $\kappa = 5$. The latter is suppressed with respect to the indirect LNV contribution of FIG. 2 by a factor

$$\frac{M_4^{LL}}{M_1} \sim \frac{p^{2}}{\Lambda^{2}}, \tag{7.33}$$

to be compared with (7.30) and (7.15).

The result of our analysis of the relative importance of different contributions to the generic $\Delta L = 2$ process $WW \longrightarrow ee$ underlying various $0\nu2\beta$-type processes may be summarized as follows: if the $\mathcal{X}$ and $\mathcal{Y}$ spurions are of comparable strength $\xi \sim \eta$, the LEET counting guarantees the dominance of the indirect LNV contribution arising merely from the neutrino Majorana mass terms. In this case, both final electrons are left-handed and the rate of LNV is essentially determined by the Majorana mass matrix. On the other hand, it is not possible to a priori exclude a hierarchy $\xi \sim \eta p/\Lambda$.

We know that $\xi\eta$ controls the scale of the top quark mass. A separate information on ξ and η can be obtained studying non-standard couplings of left-handed and right-handed fermions respectively, see Section VI. A hierarchy $\xi \sim \eta p/\Lambda$ would imply that indirect and direct LNV contributions could be of a comparable size, making problematic the use of processes such as $0\nu2\beta$ to measure the absolute size of neutrino masses. Notice however, that even in this case, the indirect Majorana mass term of FIG. 2 would dominate the emission of two left-handed charged leptons: the competing direct LNV contribution leads to the emission of one (FIG. 2) or two (FIG. 3) right-handed leptons. The answer might come from the detailed quantitative analysis of NLO in the LNV conserving sector. To conclude, we mention that, although we have performed the comparison of respective contributions to the LNV process $WW \longrightarrow ee$ in the hypothesis that $(B-L)$-breaking is described by the case labelled I earlier, the relative importance of the various terms will be unmodified if one were to analyze cases II and III.

VIII. QUASI-STERILE LIGHT RIGHT-HANDED NEUTRINOS AND COSMOLOGY

This section is to a large extent self-contained. We describe the implications of light quasi-sterile ν_R as a dark matter (DM) component, i.e. examine whether the ν_R fit in the standard cosmology, without affecting observables. We also consider the case where the ν_R would give the bulk of the DM contribution, only to conclude that this seems to be excluded if one includes the latest analyses. Before proceeding, we can only repeat the warning that our colleagues cosmologists themselves issue: when interpreting limits derived from cosmology, one should keep in mind that the analyses are carried assuming the standard cosmological model with given priors on the parameter ranges, whereas there could be more than one non-standard effect compensating each other. In Section VIII A, we give a succinct background necessary in order to follow the subsequent discussion. As a primary source for more detailed information, we hasten to recommend the reviews of [38], and the references therein. We will only quote additional references for specific points.

The ν_R we have introduced in the Higgs-less effective theory are quite different from the N_R of the see-saw extension of the SM: they are not only light, but also

stable. Indeed, the only mass terms for neutrinos are of the Majorana type, and our ν_R are characterized by the following properties: i) they are neutral, ii) their main interactions are through neutral currents with the Z^0, and are suppressed with respect to weak interactions —as parameterized by the factor η^2 in (6.26)— and, iii) they are lighter than the TeV since they are part of the LEET, and presumably, from the estimates of Sections V C 3 and VI B, lighter than 10 keV.

Note that point iii) is valid for all three possible assumptions about $(B-L)$-breaking, denoted I, II, III in Section IV B 4. Note also, regarding point ii), that, from (6.26) and bounds on the Z^0 invisible width, we deduced in Section VI B that η^2 should be smaller than about 10^{-2}. Such a bound applies only for $m_{\nu_R} \lesssim M_Z/2$, which we expect to hold, given the remark after point iii) above.

A. Relevant cosmological observations

The presence of light ν_R may impact cosmological observations, which are influenced by the composition of the universe, i.e. the respective contributions of baryonic matter, dark matter of various types, and dark energy. The first input is the Hubble constant —or its reduced version h— from correlations of luminosity and redshift for standard candles such as cepheids and type-Ia supernovæ. The latter also constrain one combination of parameters pertaining to the composition of the universe. Other measurements give constraints on various combinations of such parameters, and must be combined to resolve the degeneracies between them. One can use the power spectrum of galaxy distribution, which gives information about large-scale structure (LSS) formation, itself strongly influenced by the dark matter density. Looking further back towards the past, anisotropies of the cosmic microwave background radiation (CMB) recently helped to improve many constraints. Considering the even earlier history of the universe, we recall that the duration of the primordial nucleosynthesis (Big-Bang nucleosynthesis or BBN) is influenced by the expansion rate of the universe, itself dependent on the energy density. If the latter is modified, the observed abundance of the various elements in the galactic and intergalactic medium can be difficult to reproduce. Intergalactic gas clouds can be used not only to determine the relative abundance of different nuclides after BBN; one can also infer a density spectrum along the line of sight in this medium —from absorption by neutral hydrogen at various redshifts— which pertains to the more recent history of structure formation. This is the so-called Lyman-α forest observation. Since the density perturbations are smaller than in the case of galaxy surveys, one can use the assumption of linear growth down to smaller scales.

With the present accuracy, all these observations are well reproduced by a flat ΛCDM model in which the largest contribution to the present day density is provided in the form of a cosmological constant, representing a fraction $\Omega_\Lambda \simeq 0.7$ of the critical density. Only a fraction $\Omega_{\rm M} \simeq 0.3$ is in the form of matter, yielding a spatially flat universe —as experimentally observed with the present accuracy. Since the contribution of experimentally observed particles is known to be marginal, the ΛCDM model assumes dark matter of the cold type (CDM) to account for $\Omega_{\rm M}$. However, admixtures of other types of dark matter, or even a main contribution from warm dark matter (WDM) particles with masses of order 1 keV can yield a good fit for structure formation [51, 52].

Since our light ν_R's do not oscillate or decay to ν_L, they provide an additional source of dark matter not usually considered. In the remainder of this Section, we therefore explore the bounds on their masses and couplings that can be deduced from cosmology. This reasoning assumes that the universe was once hot enough for the ν_R to be in chemical equilibrium with the other particles via their weaker-than-weak interactions (6.26).

We also assume that the relative contributions of both the cosmological constant and the dark matter (DM) are unchanged —respectively $\Omega_\Lambda \simeq 0.7$ and $\Omega_{\rm DM} \simeq 0.3$—, but use different hypotheses as to the composition of the DM component. Depending on the strength of their coupling to the Z^0 (6.26) —i.e. on the value of the spurion parameter η—, and on their masses —i.e. depending on whether we assume the $(B-L)$-breaking option I, II or III of Section V C 3—, the ν_R contribution will generally not yield the measured DM density. Clearly, we can exclude scenarios in which the predicted density would be too high. In the other case where it is too low, it simply means that the ν_R on their own cannot explain the whole DM density: we then need to assume the presence of a CDM complement to obtain $\Omega_{\rm DM} \simeq 0.3$. We can then obtain stronger constraints by using the list of observations mentioned above.

B. Constraints on ν_R as dark matter

The ν_R's couple mainly to the Z^0, via the operator (6.26), with a coupling constant of order η^2 times the one of left-handed neutrinos. Their freeze-out temperature T_D will then be greater than that of ν_L (which is equal to a few MeV) by a factor

$$\frac{T_D}{T_D^{\nu_L}} \;\sim\; \eta^{-4/3}. \tag{8.1}$$

For reasonable values of η (see Section VI B), this yields the following range of variation for T_D

$$10\,\text{MeV} \;\lesssim\; T_D \;\lesssim\; 1\,\text{GeV}\,. \tag{8.2}$$

This temperature is obtained by equating the annihilation rate, given at temperatures below M_Z by $n_{\nu_R} \times \langle\sigma v\rangle_T \;\sim\; g^4\eta^4T^5/M_Z^4$, with the expansion rate $H \sim T^2/M_{\rm Planck}$ of the universe at that same temperature. The thermal density n_{ν_R} used assumes that the ν_R are relativistic at that temperature, i.e. $m_{\nu_R} < T_D$ which

we find is always true for the range (8.2) as long as $m_{\nu_R} \lesssim 1\,\mathrm{MeV}$, which we expect to always hold. Hence our ν_R indeed decouple while they are still relativistic, and will never yield a CDM candidate, whose mass should be above a GeV [53, 54]. Note also that in the EWSB scenario we consider, there are no new particles below a TeV. Therefore, the range (8.2) translates into a maximum number of degrees of freedom at decoupling of 247/4 [38], as in the SM at the same scale.

1. *Relic density*

Viable scenarios must first meet the condition that each particle contributing to DM produces a relic density that is smaller or equal to $\Omega_{\mathrm{DM}} \simeq 0.3$. Remember that the contribution of a particle decreases as its decoupling temperature increases [55]. Given the upper limit of 247/4 on the number of degrees of freedom at decoupling, the sum of the masses of the ν_R's has to be below about 100 eV in order to respect the bound. This upper limit would correspond to a situation where the bulk of DM is provided by the ν_R, i.e. a ΛWDM scenario. In that case we would not need any additional source of DM. However seducing such a scenario may appear, we will see in Section VIII B 4 that it now seems to be excluded by more recent measurements of smaller scale structure formation from Lyman-α forests, if we adopt such analyses.

2. *Combining CMB and LSS*

If the ν_R would decouple as late as active ν_L neutrinos do [56, 57], i.e. if they constituted hot hark matter (HDM), the upper limit on their mass as deduced from CMB and LSS would be of order the eV in order for the density fluctuations not to be erased by ν_R free-streaming. In practice this remains true if the decoupling temperature is lower than the QCD transition temperature. If η^2 is small enough that T_D is above the QCD transition, the early decoupling of the ν_R means that the decrease of the number density and temperature due to the expansion of the universe is not compensated by the subsequent annihilation of other species. Hence, the contribution of the ν_R to the energy density after the QCD transition is small with respect to that of other relativistic species. The limit from CMB and LSS on the mass of such warm dark matter (WDM) is then almost inexistent [56].

Note that data on structure formation inferred from galaxy surveys must be excluded from this analysis if they correspond to wave-numbers larger than about $0.2h\,\mathrm{Mpc}^{-1}$. The reason is that the growth of such perturbations cannot be consistently considered to occur within the linear regime. To extend the analysis and strengthen the constraints from structure formation, we will see in Section VIII B 4 that the data set may be extended by considering intergalactic gas clouds, where the density is lower.

3. *Influence on BBN*

If the ν_R's decouple before the QCD transition, their influence on BBN via the speed-up parameter (i.e. their contributions to the energy density at the epoch $0.06\,\mathrm{MeV} < T_\gamma < 1\,\mathrm{MeV}$) is reduced, by the same argument as above in Section VIII B 2 [58]. Alternatively, one may say that the equivalent number of "full-strength" neutrino species N_ν is reduced.

If the ν_R decoupled at $T_D \simeq 1\,\mathrm{GeV}$, each of them would only add 0.1 to the effective number of neutrino species. In this situation, three species of ν_R would modify the N_ν extracted from BBN within current uncertainties [59] [18]. This would not be possible if they decoupled after the QCD transition. Considering the main interactions of these ν_R (6.26), we deduce that the suppression factor η in the interaction (6.26) might need to be slightly smaller than the 0.1 mentioned earlier. However, we are talking about factors smaller than an order of magnitude here, which would be beyond the predictive power of such naive estimates from operators in the LEET: there are always unknown order-one coefficients.

4. *Constraints from Lyman-α forests*

Scenarios that satisfy constraints from LSS formation have to pass yet another test pertaining to structure formation on scales in the range $(1 \div 40)\,h^{-1}\,\mathrm{Mpc}$. This constrains particles with a long free-streaming length, i.e. hot or warm dark matter. Such constraints come from the observation of Lyman-α forests, whose interpretation has been controversial in the past, but is becoming more and more widely accepted these days.

One can guess that there should be a lower bound on the mass if the properties for structure formation have to mimic those of CDM [61], and avoid the destructive effect of free-streaming. This is true for the case where a large proportion of the DM comes from this WDM. The bound coming from the study of Lyman-α forests is in this case that the mass of the WDM particle should be higher than about 500 eV [62, 63] [19]. There would then be no

[18] The situation is at present unclear: according to some studies, even the standard BBN might have difficulties, due to tension between the abundances of various elements. See for instance the references in [38, 60].

[19] There are also claims of a conflict with the early reionization deduced from CMB measurements by the WMAP satellite, even for masses of order 10 keV [64]. However, the relation between the measured optical depth and the inferred redshift of reionization is not so direct [65, 66]. At any rate, this concern is irrelevant for the present analysis: even though we started with a range of

overlap with the upper bound of about 100 eV described in Section VIII B 1.

There is however another alternative, where most of the DM is in the form of unknown CDM, and the ν_R only provide a marginal fraction of the total density —as the ν_L's do. Remember that for the case of ν_L, the upper limit from CMB and LSS formation was of order 1 eV. This limit for ν_L is not much modified by the adjunction of Lyman-α forest results [67].

The difference with the ν_L is that our ν_R a priori decouple earlier, viz. (8.1): Section VIII B 3 indicated decoupling of ν_R before the QCD transition —in this case the bound from CMB and LSS (Section VIII B 2) was almost inexistent. The inclusion of Lyman-α forest data, if we adopt it, changes this picture drastically, and yields an upper bound for our ν_R of about 10 eV [63, 67]. This disfavors the choice III for the $(B-L)$-breaking spurion $\mathcal{Z}$ (4.59) because of relation (5.38), but leaves room for the two other options.

C. Conclusions on the different scenarios I, II, III for $(B-L)$-breaking

The allowed range for the parameters are thus: i) suppression factor η^2 with respect to weak interactions of about 10^{-2} or maybe smaller and, ii) ν_R masses possibly up to 10 eV, or smaller. Of the different scenarios for $(B-L)$-breaking, only the choices labeled as I and II seem compatible with this limit on the masses. In particular, the choice I, which we have adopted for our main line of discussion, seems safe with respect to current cosmological constraints.

Let us also mention the possibility studied in [68], and references therein, in which keV sterile neutrinos are populated via oscillations with the active ones: such a situation cannot occur here if we assume the ν_R sign-flip —the $\mathbb{Z}_2$ symmetry of (5.26)— to be unbroken.

IX. SUMMARY AND CONCLUSIONS

i) Low-energy effective theories of EWSB without a Higgs in which deviations from the SM would occur at leading order are untenable from the phenomenological standpoint. A Higgs-less LEET based on the symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and operating with naturally light gauge bosons and chiral fermions unavoidably suffers from this disease: at the leading order $\mathcal{O}(p^2)$, the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry allows for non-standard oblique corrections, non-standard fermion gauge boson vertices and —last but not least— unsuppressed $\Delta L = 2$ Majorana masses for left-handed neutrinos. In the SM all these operators would correspond to dimensionally suppressed operators containing at least two powers of the elementary Higgs field. Consequently, any Higgs-less scenario must first of all address the question of natural suppression of non-standard operators at the leading order.

ii) We have shown that non-standard $\mathcal{O}(p^2)$ vertices are suppressed if the LEET is based from the onset on a non-linearly realized higher symmetry $S_{\rm nat} \supset S_{\rm red} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, generalizing the concept of custodial symmetry. $S_{\rm nat}$ turns out to represent a hidden symmetry of the Higgs-less vertices of the SM Lagrangian itself in the limit of vanishing fermion masses. The manifestation of $S_{\rm nat}$ is precisely the absence of non-standard $\mathcal{O}(p^2)$ vertices which would be allowed by $S_{\rm red}$ alone. $S_{\rm nat}$ corresponds to the maximal linear local symmetry the theory could have if its symmetry breaking sector (Goldstone bosons) would be decoupled from the gauge/fermion sector.

iii) In the LEET with a minimal light particle content one has $S_{\rm nat} = [\mathrm{SU}(2) \times \mathrm{SU}(2)]^2 \times \mathrm{U}(1)_{B-L}$. The set of variables on which $S_{\rm nat}$ acts linearly would thus involve thirteen gauge connections. The gauge and symmetry breaking sectors are coupled and the redundant fields are eliminated through constraints invariant under $S_{\rm nat}$. These constraints leave us with the four gauge fields of the SM on which only the subgroup $S_{\rm red} \subset S_{\rm nat}$ acts linearly, and with a set of three non-propagating spurion fields transforming non trivially under $S_{\rm nat}$. The very existence and properties of spurions are not a matter of choice but follow from the covariant reduction of the symmetry $S_{\rm nat}$ to its subgroup $S_{\rm red}$.

iv) Spurions are an integral part of the theory. They are crucial to maintain the covariance under $S_{\rm nat}$ even if the latter is explicitly broken. There exists a gauge in which spurions reduce to three real constants ξ, η, ζ, which are used as small expansion parameters introducing a technically natural hierarchy of symmetry breaking effects. Among the spurions, there is necessarily one carrying two units of the $B-L$ charge: it arises from the reduction of the right handed isospin and $\mathrm{U}(1)_{B-L}$ subgroup of $S_{\rm nat}$ to $\mathrm{U}(1)_Y$ symmetry of the SM. Consequently, the spurion formalism unavoidably predicts $\Delta L = 2$ vertices. Their order of magnitude is controlled by a single spurion parameter ζ^2. The latter is postulated to be much smaller than ξ and η which parametrize the lepton-number conserving symmetry-breaking effects.

v) The effective Lagrangian is constructed and renormalized order by order in a double expansion: according to the infrared dimensions —c.f. equation (3.21)— and according to powers of spurions. At each order all operators invariant under $S_{\rm nat}$ have to be included. At the leading order $\mathcal{O}(p^2)$ with no spurion insertion, one recovers the SM interaction vertices between massive $W^\pm, Z^0$ and massless fermions. As in the SM, $W^\pm$ and Z^0 acquire their masses by the Higgs mechanism involving the three GBs. On the other hand, Dirac fermion masses are

possible masses for ν_R extending up to 10 keV, we have seen in Section *VIII B* 1 that, in our particular case, we could exclude $m_{\nu_R} \gtrsim 100\,\mathrm{eV}$ since otherwise the decoupling could not occur early enough as to yield $\Omega_{\nu_R} \lesssim 0.3$.

necessarily suppressed by (at least) the spurion factor $\xi\eta$. This fact allows to extend the Weinberg's power counting to fermions as discussed in Section III A 3: it suggests the counting rule $\xi\eta = \mathcal{O}(p)$.

vi) At higher spurion orders one finds all the non-standard couplings encountered before (Sect III C), but now suppressed by definite spurion factors dictated by symmetry properties. The spurion formalism thus suggests a hierarchy between possible effects beyond the SM. This might indicate new directions in the search of the latter. For instance, in the lepton-number conserving sector, we have concentrated on $\mathcal{O}\left(p^2\right)$ vertices containing at most two powers of spurions ξ or η. Such non-standard vertices represent universal modifications of both left-handed and right-handed fermion couplings. They are suppressed with respect to the leading-order SM contributions but, according to the power counting, they should be more important than the loop contributions. The complete list of these vertices is displayed in Section VI. They exhaust all non-standard lepton-number conserving effects which arise in our LEET at the NLO. Let us stress that the oblique corrections only arise at the NNLO and cannot be disentangled from the loops. This is consistent with the absence of observation of oblique corrections beyond the SM.

vii) As already stressed, the LNV sector is to a large extent determined by the framework and by the symmetry properties of the LEET. First, in order to allow the hidden custodial symmetry to be at work at energies at which the LEET is relevant, the light rightshanded neutrinos must be present as doublet partners of right handed electrons. At the leading order ν_R's decouple from all gauge fields (as in the SM), but have a residual interaction with Z^0, suppressed with respect to the SM coupling by the spurion factor η^2 . This allows one to assume that the discrete sign flip symmetry $\nu_R \longmapsto -\nu_R$ (not spoiled by anomalies) remains exact to all orders of the LEET. This forbids to all orders any neutrino Dirac mass term as well as charged leptonic right-handed currents. Both left-handed and right-handed neutrinos (Majorana) masses m_{ν_L} and m_{ν_R} are suppressed by the spurion factor ζ^2. They nevertheless differ, due to different contributions of spurions ξ and η. This represents an alternative to the see-saw mechanism. With the existing constraints on the mass of active neutrinos and naive estimates of spurion factors, one infers that the heaviest ν_R can hardly be heavier than 10 keV.

viii) We have reviewed pertinent cosmological constraints on super-weakly interacting particles, coming from the observations of the CMB, of structure formation and BBN. This leaves open the possibility of $m_{\nu_R} \lesssim 10\,\mathrm{eV}$, which satisfies all constraints in the standard Λ CDM cosmology. Either of the two scenarios I and II for $B-L$ breaking fall in this parameter space, and are therefore allowed. The situation would be different for scenario III, with a heavier ν_R around the keV which would provide the bulk of the DM in a Λ WDM cosmology. The recent interpretation of Lyman-α data for smaller-scale structure formation, if taken at face value, would exclude this theoretically well-motivated scenario.

ix) Finally we have analyzed the LO and NLO direct LNV vertices which may compete with the indirect contribution of Majorana masses to the process $W^-W^- \longrightarrow e^-e^-$, a building block for $0\nu2\beta$ decay. We classify, independently of the LEET, the different types of contributions —usually omitted in the literature— besides the indirect LNV. Note that this could a priori invalidate the extraction of the absolute neutrino mass scale from $0\nu2\beta$ decay. We then show that the LEET provides estimates of their respective magnitudes: we find that the connection between Majorana masses and $0\nu2\beta$ decay holds, up to corrections in powers of spurions. Of course, such corrections might still turn out to be large.

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APPENDIX A: PROOF OF THE GENERALIZED WEINBERG POWER-COUNTING FORMULA

Here we present a derivation of the power-counting formula (3.28) in the presence of gauge fields and chiral fermions, as described in Section III A. We start with the identity that simply counts the powers of t coming from the rescalings (3.22-3.26) in a loop integral

$$d_{\mathrm{IR}}\left[\Gamma\right] \;=\; N_\partial + N_g + \frac{1}{2}N_f^{\mathrm{ext}} - 2I_b - I_f + 4L. \quad \text{(A1)}$$

Here, N_∂ and N_g are respectively the total number of derivatives and of coupling constants that are contained in the vertices $1, \cdots, V$ of the diagram Γ. N_f^{ext} is the total number of external (non-contracted) fermion fields. I_b and I_f denote the number of internal boson and fermion lines. The last term $4L$ accounts for the L-loop momentum integral. One first observes that

$$I_f \;=\; \frac{1}{2}\sum_{v=1}^{V} n_f^{\mathrm{int}}\left[\mathcal{O}_v\right], \quad \text{(A2)}$$

where $n_f^{\mathrm{int}}\left[\mathcal{O}_v\right]$ is the number of internal fermion lines ending up in the vertex $\mathcal{O}$. Writing

$$N_f^{\mathrm{ext}} \;=\; \sum_{v=1}^{V} n_f^{\mathrm{ext}}\left[\mathcal{O}_v\right], \quad \text{(A3)}$$

where $n_f^{\rm ext}[\mathcal{O}_v]$ stands for the number of external fermion lines ending up in the vertex $\mathcal{O}$, one finds that

$$2I_f + N_f^{\rm ext} \;=\; \sum_{v=1}^{V} n_f[\mathcal{O}_v]\,, \qquad \text{(A4)}$$

where $n_f[\mathcal{O}_v] = n_f^{\rm int}[\mathcal{O}_v] + n_f^{\rm ext}[\mathcal{O}_v]$ represents the number of fermion fields in the vertex $\mathcal{O}$. Consequently, (A1) can be written as

$$\begin{aligned} d_{\rm IR}[\Gamma] \;&=\; 4L - 2(I_b + I_f) \\ &+ \sum_{v=1}^{V}\left(n_\partial[\mathcal{O}_v] + n_g[\mathcal{O}_v] + \frac{1}{2}n_f[\mathcal{O}_v]\right), \end{aligned} \qquad \text{(A5)}$$

where the infrared dimension $d_{\rm IR}[\mathcal{O}_v]$ (3.21) of each vertex $\mathcal{O}_v$ appears. It remains to use the identity

$$I_b + I_f \;=\; V + L - 1, \qquad \text{(A6)}$$

and rewrite (A5) as the original Weinberg formula (3.28)

$$d_{\rm IR}[\Gamma] \;=\; 2 + 2L + \sum_{v=1}^{V}\left(d_{\rm IR}[\mathcal{O}_v] - 2\right), \qquad \text{(A7)}$$

which now holds beyond the framework of χPT. It is worth stressing that the validity of (A7) in the presence of light fermions is tied to the assignment of infrared dimension $d_{\rm IR}[f] = 1/2$ to chiral fermion fields. For a general $d_{\rm IR}[f]$, the third term in the right-hand side of (A1) would be modified to $d_{\rm IR}[f]\,N_f^{\rm ext}$, whereas (A2) would remain unchanged. Consequently, unless $d_{\rm IR}[f] = 1/2$, a suitably modified (A4) does not help in eliminating I_f and $N_f^{\rm ext}$ in terms of L and the total number of fermion fields in the vertices $\sum n_f[\mathcal{O}_v]$.

A last remark about the power-counting formula in a space-time of dimension D could be of interest. Note first that the mass-dimension of fields, coupling constants g, and the GB coupling f depends on D, whereas the infrared dimension of all these quantities as introduced in section III A does *not*. The only modification of the power counting formula (A9) thus arises from the loop integrals: the last term in (A1) becomes DL instead of $4L$. Therefore with the definition

$$\Delta[\Gamma] \;\equiv\; \sum_{v=1}^{V}\left(d_{\rm IR}[\mathcal{O}_v] - 2\right) \;\geqslant\; 0, \qquad \text{(A8)}$$

the final formula reads

$$d_{\rm IR}[\Gamma] \;=\; 2 + (D-2)L + \Delta[\Gamma], \qquad \text{(A9)}$$

showing explicitly the improved low-energy suppression of loops as D increases. Whereas higher dimensional (gauge) theories are too UV singular to admit an expansion in powers of coupling constant only, they might still be a privileged arena for LEETs formulated as a systematic low-energy expansion.

APPENDIX B: UNITARITY ORDER BY ORDER

The Weinberg power-counting formula (A9) allows one to compare how the exact unitary S-matrix is successively approximated in a renormalized expansion in powers of the coupling constant, and in the low-energy expansion. To this end, we consider a transition matrix element $T_{i\to f}$ where the initial (final) state contains a set of i (f) particles. We first recall the unitary condition, which reads

$$\operatorname{Im} T_{i\to f} \;=\; \sum_{n\geqslant 2}\int {\rm d}\{n\}\,(T_{f\to n})^{*}\,T_{i\to n}, \qquad \text{(B1)}$$

where $\int {\rm d}\{n\}$ stands for the integral over the Lorentz invariant phase space of an n-particle intermediate state

$$\begin{aligned} \int {\rm d}\{n\} \;&\propto\; \int\left(\prod_{i=1}^{n} {\rm d}^D p_i\,\delta\left(p_i^2 - m_i^2\right)\theta\left(p_i^0\right)\right) \\ &\times \delta^D\left(P - \sum_{j=1}^{n} p_j\right). \end{aligned} \qquad \text{(B2)}$$

We first deal with a renormalizable theory with trilinear $\sim g$ and quadratic $\sim g^2$ couplings. The amplitude is approximated by a series in g, where the power of g appearing in a diagram with a given i and f, only depends on the number of loops L

$$T_{i\to f} \;=\; \sum_{L\geqslant 0} T^{L}_{i\to f}, \qquad \text{(B3)}$$

where $T^L_{i\to f}$ is of degree $2L+i+f-2$ in g. At a given order in g, the condition (B1) becomes an identity between a finite number of Feynman diagrams

$$\begin{aligned} \operatorname{Im} T^{L}_{i\to f} \;&=\; \sum_{n\geqslant 2}^{L+1}\sum_{l,l'\geqslant 0}\delta_{l+l'+n,L+1} \\ &\times \int {\rm d}\{n\}\left(T^{l}_{f\to n}\right)^{*} T^{l'}_{i\to n}. \end{aligned} \qquad \text{(B4)}$$

A few comments are in order:

i) As long as the theory is properly renormalized, the expected identity (B4) automatically follows from the cutting rules of Feynman diagrams. In principle, it holds at all energies. This however, does not give any information on the size of violations of the unitary condition (B1) for a finite energy and a given truncation of the expansion (B3).

ii) Equation (B4) is a recurrence relation. If one knows all amplitudes T^l for $l = 1,\cdots,L-1$, it is possible to compute $\operatorname{Im} T^L_{i\to f}$ in all channels. $\operatorname{Re} T^L$ is then, in principle, determined by analyticity, up to a real subtraction polynomial. Requiring renormalizability order by order is a restriction on the degree of these polynomials, and thereby on the high-energy behavior of $\operatorname{Im} T^L_{i\to f}$.

iii) $\mathrm{Im}\, T^0 = 0$, hence tree amplitudes violate unitarity as long as $\mathrm{Re}\, T^0 \neq 0$. This fact often motivates searches for additional light tree-level exchanges (scalars, KK excited states) which would keep $\mathrm{Re}\, T^0$ small for energies as large as possible. Experiments will tell us whether such a situation, allowing for a perturbative expansion, is realized in the real world.

We now turn to the low-energy expansion, in which one expands the amplitudes as

$$T_{i\to f} \;=\; \sum_{d_{\mathrm{IR}}\geqslant 2} t^{d_{\mathrm{IR}}}_{i\to f}, \tag{B5}$$

where d_{IR} is the homogeneous degree of $t^{d_{\mathrm{IR}}}_{i\to f}$. The analysis of the unitary condition (B1) order by order in the low-energy expansion (B5) requires the knowledge of the infrared dimension of $\int \mathrm{d}\{n\}$ (B2). Assuming that all particles in the intermediate states are naturally light, i.e. their masses are suppressed as at least $\mathcal{O}(p)$, one infers the following *suppression at low-energies* of their Lorentz-invariant phase-space

$$\int \mathrm{d}\{n\} \;=\; \mathcal{O}\left(p^{(D-2)(n-1)-2}\right). \tag{B6}$$

It is crucial for the remainder that the creation of many-particle states is suppressed [20]. One then isolates contributions of infrared dimension d_{IR} in (B1)

$$\mathrm{Im}\, t^{d_{\mathrm{IR}}}_{i\to f} \;=\; \sum_{n\geqslant 2}^{\left[\frac{d_{\mathrm{IR}}-2}{D-2}\right]+1} \sum_{d,d'\geqslant 2} \delta_{d+d'+(D-2)(n-1),d_{\mathrm{IR}}+2} \times \int \mathrm{d}\{n\} \left(t^{d}_{f\to n}\right)^* t^{d'}_{i\to n}. \tag{B7}$$

In this low-energy expansion, the connection between the degree of a diagram and the number of loops is more involved than following (B3). It is therefore useful to introduce Feynman amplitudes $M^L_{i\to f}\{\Delta\}$ collecting L-loop diagrams and to use the sum (A8) $\Delta = \sum_v (d_{\mathrm{IR}}[\mathcal{O}_v] - 2)$ over all vertices $\mathcal{O}$ involved, such that

$$M^L_{i\to f}\{\Delta\} \;=\; \mathcal{O}\left(p^{2+(D-2)L+\Delta}\right). \tag{B8}$$

We decompose $t^{d_{\mathrm{IR}}}_{i\to f}$ as a sum of such collections of Feynman diagrams, writing

$$t^{d_{\mathrm{IR}}}_{i\to f} \;=\; \sum_{L=0}^{\left[\frac{d_{\mathrm{IR}}-2}{D-2}\right]} M^L_{i\to f}\{\Delta = d_{\mathrm{IR}} - (D-2)L - 2\}, \tag{B9}$$

where we have taken into account that, for each term $M^L_{i\to f}\{\Delta\}$ to contribute to the order d_{IR}, the value of Δ is fixed by the number of loops to be $\Delta = d_{\mathrm{IR}} - (D-2)L - 2$. Also, the the upper limit $L \leqslant \left[\frac{d_{\mathrm{IR}}-2}{D-2}\right]$ comes from the fact that $\Delta \geqslant 0$.

In a LEET, renormalization is based on a cancellation of divergences between terms of the finite sum (B9) with different L but which carry the same power d_{IR}. In other words, divergences of multi-loop diagrams containing low infrared dimension vertices are absorbed by diagrams with fewer loops, but higher dimension vertices [69]. This procedure is repeated for every fixed $d_{\mathrm{IR}} \geqslant 2$. For $d_{\mathrm{IR}} = 2$ and $d_{\mathrm{IR}} = 3$, $t^{d_{\mathrm{IR}}}$ does not involve loops. It is merely given by (finite) tree diagrams with $\Delta = 0$ and $\Delta = 1$. Accordingly, $\mathrm{Im}\, t^{2,3} = 0$, in agreement with the unitarity condition (B7).

It is instructive to compare the SM L-loop amplitude $T^L_{i\to f}$ with the Feynman amplitudes $M^L_{i\to f}\{\Delta\}$ of the minimal Higgs-less LEET. Since the $\mathcal{O}(p^2)$ vertices of the LEET coincide with those of the SM without the Higgs fields, the term with $\Delta = 0$ in $t^{d_{\mathrm{IR}}}_{i\to f}$ (B9) is identical to the collection of L-loop SM diagrams that do not contain internal Higgs lines. Symbolically

$$T^L_{i\to f} \;=\; T^L_{i\to f}|_{\text{Higgs exchange}} + M^L_{i\to f}\{\Delta = 0\}. \tag{B10}$$

Similarly, the LEET unitarity relation (B7) does not include physical Higgs particles in the intermediate states $\{n\}$, whereas the SM unitarity relation (B4) does. The role of (virtual) Higgs contributions in the renormalization and unitarization of the SM is replaced in the Higgs-less LEET by effects of higher dimension vertices $\Delta > 0$ in (B9). After renormalization, equation (B9) can be rewritten in terms of renormalized amplitudes $\hat{M}^L_{i\to f}\{\Delta, \mu\}$

[20] As in Appendix A, the suppression gets more effective as the space-time dimension D grows.

$$t_{i\to f}^{d_{\rm IR}} \;=\; \sum_{L=0}^{\left[\frac{d_{\rm IR}-2}{D-2}\right]} \hat{M}_{i\to f}^{L}\left\{\Delta = d_{\rm IR} - (D-2)\,L - 2, \mu\right\}. \tag{B11}$$

In this equation, the individual terms $\hat{M}_{i\to f}^{L}\{\Delta,\mu\}$ depend on a renormalization scale μ (introduced e.g. via dimensional regularization and renormalization) whereas the whole sum (B11), i.e. the physical quantity $t_{i\to f}^{d_{\rm IR}}$, is both finite and μ-independent [69]. The recurrence unitarity relation (B7) then follows from the cutting formula for the renormalized one-loop amplitudes $\hat{M}_{i\to f}^{L}\{\Delta,\mu\}$

$$\mathrm{Im}\, M_{i\to f}^{L}\{\Delta,\mu\} \;=\; \sum_{n\geqslant 2}^{L+1}\sum_{\delta,\delta'\geqslant 0}\delta_{\delta+\delta',\Delta}\sum_{l,l'\geqslant 0}\delta_{l+l'+n,L+1}\int \mathrm{d}\{n\}\left(\hat{M}_{f\to n}^{l}\{\delta,\mu\}\right)^{*}\hat{M}_{i\to n}^{l'}\{\delta',\mu\}. \tag{B12}$$

This equation may be viewed as an extension of the cutting identity (B4) for the case where one has to take into account vertices with different dimensions. It should hold independently of the renormalization scale μ.

This analysis of course does not say much about the actual size of violations of unitarity if the LEET (B5) is truncated at a given order $d_{\rm IR}$. By the very definition of the LEET, we can expect that for a given $d_{\rm IR}$, the violation of unitarity will increase with energy. On the other hand, as demonstrated for the case of χPT in $\pi\pi$ scattering comparing $\mathcal{O}\left(p^4\right)$ and $\mathcal{O}\left(p^6\right)$ —see FIG. 1 of [70] —, one may expect that the energy up to which unitarity is satisfied with a given precision increases with $d_{\rm IR}$.

[1] G. 't Hooft, in [71].
[2] S. Weinberg, Physica **A96**, 327 (1979).
[3] J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984).
[4] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[5] T. Appelquist, M. J. Bowick, E. Cohler, and A. I. Hauser, Phys. Rev. **D31**, 1676 (1985).
[6] B. Holdom and J. Terning, Phys. Lett. **B247**, 88 (1990).
[7] H. Georgi, Nucl. Phys. **B363**, 301 (1991).
[8] D. Espriu and M. J. Herrero, Nucl. Phys. **B373**, 117 (1992).
[9] F. Feruglio, Int. J. Mod. Phys. **A8**, 4937 (1993), hep-ph/9301281.
[10] J. Wudka, Int. J. Mod. Phys. **A9**, 2301 (1994), hep-ph/9406205.
[11] E. Bagan, D. Espriu, and J. Manzano, Phys. Rev. **D60**, 114035 (1999), hep-ph/9809237.
[12] A. Nyffeler and A. Schenk, Phys. Rev. **D62**, 113006 (2000), hep-ph/9907294.
[13] A. Nyffeler (1999), hep-ph/9912472.
[14] J. Hirn and J. Stern, Eur. Phys. J. **C34**, 447 (2004), hep-ph/0401032.
[15] M. E. Peskin and T. Takeuchi, Phys. Rev. **D46**, 381 (1992).
[16] A. C. Longhitano, Phys. Rev. **D22**, 1166 (1980).
[17] R. D. Peccei and X. Zhang, Nucl. Phys. **B337**, 269 (1990).
[18] G. Senjanovic and R. N. Mohapatra, Phys. Rev. **D12**, 1502 (1975).
[19] P. Sikivie, L. Susskind, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. **B173**, 189 (1980).
[20] J. Hirn and J. Stern, JHEP **09**, 058 (2004), hep-ph/0403017.
[21] G. Cacciapaglia, C. Csaki, C. Grojean, and J. Terning, Phys. Rev. **D71**, 035015 (2005), hep-ph/0409126.
[22] V. Bernard, M. Oertel, E. Passemar, and J. Stern (in preparation).
[23] P. Minkowski, Phys. Lett. **B67**, 421 (1977).
[24] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[25] M. Gell-Mann, P. Ramond, and R. Slansky, in [72].
[26] T. Yanagida, in [73].
[27] T. Appelquist and C. Bernard, Phys. Rev. **D22**, 200 (1980).
[28] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
[29] R. Urech, Nucl. Phys. **B433**, 234 (1995), hep-ph/9405341.
[30] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[31] H. Georgi, *Weak interactions and modern particle theory* (Benjamin/Cummings Publishing, 1984).
[32] H. Georgi, Nucl. Phys. **B331**, 311 (1990).
[33] C. P. Burgess and D. London, Phys. Rev. **D48**, 4337 (1993), hep-ph/9203216.
[34] H. Georgi, Ann. Rev. Nucl. Part. Sci. **43**, 209 (1993).
[35] A. Pich, in [74], hep-ph/9806303.

[36] H. J. He, Y. P. Kuang, and C. P. Yuan, Phys. Rev. **D55**, 3038 (1997), hep-ph/9611316.
[37] http://lepewwg.web.cern.ch/LEPEWWG/.
[38] S. Eidelman et al. (Particle Data Group), Phys. Lett. **B592**, 1 (2004), http://pdg.lbl.gov/.
[39] C. Arzt, M. B. Einhorn, and J. Wudka, Phys. Rev. **D49**, 1370 (1994), hep-ph/9304206.
[40] M. B. Einhorn and J. Wudka, Phys. Rev. Lett. **87**, 071805 (2001), hep-ph/0103034.
[41] C. Csaki, C. Grojean, L. Pilo, and J. Terning, Phys. Rev. Lett. **92**, 101802 (2004), hep-ph/0308038.
[42] K. Fujikawa (2004), hep-ph/0407331.
[43] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002), hep-ph/0207036.
[44] A. Aguilar et al. (LSND), Phys. Rev. **D64**, 112007 (2001), hep-ex/0104049.
[45] LEP Collaborations (2003), hep-ex/0312023.
[46] R. S. Chivukula, E. H. Simmons, H.-J. He, M. Kurachi, and M. Tanabashi, Phys. Lett. **B603**, 210 (2004), hep-ph/0408262.
[47] R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, Nucl. Phys. **B703**, 127 (2004), hep-ph/0405040.
[48] R. Foadi, S. Gopalakrishna, and C. Schmidt, Phys. Lett. **B606**, 157 (2005), hep-ph/0409266.
[49] G. Sánchez-Colón and J. Wudka, Phys. Lett. **B432**, 383 (1998), hep-ph/9805366.
[50] A. Atre, V. Barger, and T. Han (2005), hep-ph/0502163.
[51] P. Colin, V. Avila-Reese, and O. Valenzuela, Astrophys. J. **542**, 622 (2000), astro-ph/0004115.
[52] P. Bode, J. P. Ostriker, and N. Turok, Astrophys. J. **556**, 93 (2001), astro-ph/0010389.
[53] B. W. Lee and S. Weinberg, Phys. Rev. Lett. **39**, 165 (1977).
[54] E. W. Kolb and K. A. Olive, Phys. Rev. **D33**, 1202 (1986).
[55] K. A. Olive and M. S. Turner, Phys. Rev. **D25**, 213 (1982).
[56] S. Hannestad and G. Raffelt, JCAP **0404**, 008 (2004), hep-ph/0312154.
[57] P. Crotty, J. Lesgourgues, and S. Pastor, Phys. Rev. **D69**, 123007 (2004), hep-ph/0402049.
[58] G. Steigman, K. A. Olive, and D. N. Schramm, Phys. Rev. Lett. **43**, 239 (1979).
[59] K. A. Olive, G. Steigman, and T. P. Walker, Phys. Rept. **333**, 389 (2000), astro-ph/9905320.
[60] S. Pastor (2003), hep-ph/0306233.
[61] S. Colombi, S. Dodelson, and L. M. Widrow, Astrophys. J. **458**, 1 (1996), astro-ph/9505029.
[62] V. K. Narayanan, D. N. Spergel, R. Dave, and C.-P. Ma, Astrophys. J. **543**, L103 (2000), astro-ph/0005095.
[63] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese, and A. Riotto (2005), astro-ph/0501562.
[64] N. Yoshida, A. Sokasian, L. Hernquist, and V. Springel, Astrophys. J. **591**, L1 (2003), astro-ph/0303622.
[65] S. H. Hansen and Z. Haiman, Astrophys. J. **600**, 26 (2004), astro-ph/0305126.
[66] N. Y. Gnedin, Astrophys. J. **610**, 9 (2004), astro-ph/0403699.
[67] S. Hannestad (2004), hep-ph/0409108.
[68] S. H. Hansen, J. Lesgourgues, S. Pastor, and J. Silk, Mon. Not. Roy. Astron. Soc. **333**, 544 (2002), astro-ph/0106108.
[69] J. Bijnens, G. Colangelo, and G. Ecker, Annals Phys. **280**, 100 (2000), hep-ph/9907333.
[70] M. Knecht, B. Moussallam, J. Stern, and N. H. Fuchs, Nucl. Phys. **B457**, 513 (1995), hep-ph/9507319.
[71] G. 't Hooft et al., eds., *Recent developments in gauge theories* (Plenum Press, 1980).
[72] P. Van Nieuwenhuizen and D. Z. Freedman, eds., *Supergravity* (North-Holland Publishing, 1979).
[73] O. Sawada and A. Sugamoto, eds., *Proceedings of the workshop on the unified theory and the baryon number in the universe, KEK, Japan, 1979* (1979).
[74] R. Gupta, A. Morel, E. de Rafael, and F. David, eds., *Probing the Standard Model of particle interactions* (North-Holland Publishing, 1999).